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A Parameter Estimation Subroutine Package

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16. Abstract

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PREFACE

The work described in this report was performed by the Systems Division of the Jet Propulsion Laboratory.

AKNOWLEDGEMENT

The construction of this estimation subroutine package (ESP) was motivated by an involvement with a particular problem; construction of fast, efficient and simple least squares data processing algorithms to be used for determining ephemeris corrections. Discussions with T. Duxbury led to the proposal of a subroutine strategy which would have great flexibility. The general utility of such a subroutine package was made evident by H. Koble and N. Mottinger who had a different but related problem that involved combining estimates from different missions. Thanks and credit are also due to J. Ellis, N. Hamata, and F. Peters for contributing to and experimenting with this package of subroutines.

ABSTRACT

Linear least squares estimation and regression analyses continue to play a major role in orbit determination and related areas. In this report we document a library of FORTRAN subroutines that have been developed to facilitate analyses of a variety of parameter estimation problems. Our purpose is to present an easy to use multi-purpose set of algorithms that are reasonably efficient and which use a minimal amount of computer storage. Subroutine inputs, outputs, usage and listings are given, along with examples of how these routines can be used. The following outline indicates the scope of this report: Section I, introduction with reference to background material; Section II, examples and applications; Section III, a subroutine directory summary; Section IV, the subroutine directory user description with input, output and usage explained; and Section V, subroutine FORTRAN listings. The routines are compact and efficient and are far superior to the normal equation data processing algorithms that are often used for least squares analyses.

CONTENTS

I. Introduction

Techniques related to least squares parameter estimation play a prominent role in orbit determination and related analyses. Numerical and algorithmic aspects of least squares computation are documented in the excellent reference work by Lawson and Hanson, Ref. [1]. Their algorithms, available from the JPL subroutine library, Ref. [2], are very reliable and general. Experience has, however, shown that in reasonably well posed problems one can streamline the least squares algorithm codes and reduce both storage and computer times. In this report, we document a collection of subroutines most of which we have written that can be used to solve a variety of parameter estimation problems.

The algorithms for the most part involve triangular and/or symmetric matrices and to reduce storage requirements these are stored in vector form, e.g., an upper triangular matrix U is written as

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} & \\ & U_{22} & U_{23} & U_{24} & \text{etc.} \\ & & U_{33} & U_{34} & \\ \bigcirc & & & U_{44} & \end{bmatrix} = \begin{bmatrix} U(1) & U(2) & U(4) & U(7) & \\ & U(3) & U(5) & U(8) & \text{etc.} \\ & & U(6) & U(9) & \\ \bigcirc & & & U(10) & \end{bmatrix}$$

Thus, the element from row i and column j of U, $i \leq j$, is stored in vector component $j(j-1)/2 + i$. We hasten to point out that the engineer, with few exceptions, need have no direct contact with the vector subscripting. By this we mean that the vector subscript related operations are internal to the subroutines, vector arrays transmitted from one

subroutine to another are compatible, and vector arrays displayed using the print subroutine TRIMAT appear in a triangular matrix format.

Aside: The most notable exception is that matrix problems are generally formulated using doubly subscripted arrays. Transforming a double subscripted symmetric or upper triangular matrix A(·,·) to a vector stored form, U(·) is quite simply accomplished in FORTRAN via

```
      IJ = 0
      DO 1 J = 1,N
      DO 1 I = 1,J
      IJ = IJ+1
    1 U(IJ) = A(I,J)
```

Similarly, transforming an initial vector D(·) of diagonal positions of a vector stored form, U(·), is accomplished using

```
      JJ = 0                          JJ = N*(N+1)/2
      DO 1 J = 1,N          or        DO 1 J = N,1,-1
      JJ = JJ+J                       U(JJ) = D(J)
    1 U(JJ) = D(J)                  1 JJ = JJ-J
```

The conversion on the right has the modest advantage that D and U can share common storage (i.e., U can overwrite D). These conversions are too brief to be efficiently used as subroutines. It seems that when such conversions are needed one can readily include them as in line code.

End of Aside

Although this package of subroutines is designed in the main, for the analysis of parameter estimation problems one can use it to solve problems that involve process noise. With modest amounts of additional programming one can even apply our package to filtering problems that involve colored noise and mapping. In the latter case, however, reductions gained from our use of vector storage are for the most part lost.

Mathematical background regarding Householder orthogonal transformations for least squares analyses and U-D matrix factorization for covariance matrix analyses are discussed in references [1] and [3]. Our plan is to illustrate, in Section II, with examples how one can use the basic algorithms and matrix manipulation to solve a variety of important problems. The subroutines which comprise our estimation subroutine package are described in Section III, and detailed input/output descriptions are presented in Section IV.

Section V contains FORTRAN listings of the subroutines. There are several reasons for including such listings. Making these listings available to the engineer analyst allows him to assess algorithm complexity for himself; and to appreciate the simplicity of the routines he tends otherwise to use as a black box. The routines are not truly portable, and users can, when necessary make modifications so that the subroutine package can operate on systems other than the UNIVAC 1108. When estimation problems arise to which our package does not directly apply (or which can be made to apply by an awkward concatenation of the routines) one may be able to modify the codes and widen still further the class of problems that can be efficiently solved.

II. APPLICATIONS AND EXAMPLES

Our purpose in this section is to illustrate, with a number of examples, some of the problems that can be solved using this ESP. The examples, in addition, serve to catalogue certain estimation techniques that are quite useful.

To begin, let us catalogue the subroutines that comprise the ESP:

1)	AGTRN	(A G Turner)	Agee-Turner rank 1 update
2)	A2A1	(A to A one)	Matrix A to matrix A1
3)	COMBO	(combo)	Combine R and A namelists
4)	COV2RI	(cov to R I)	Covariance to R inverse
5)	COV2UD	(cov to U D)	Covariance to U-D factors
6)	C2C	(C to C)	Permute the rows and columns of matrix C
7)	INF2R	(inf to R)	Information matrix to (triangular) R
8)	PERMUT	(permute)	Permute the columns of matrix A
9)	RINCON	(rin con)	R inverse with condition number bound
10)	RI2COV	(R I to cov)	R inverse to covariance
11)	R2A	(R to A)	Triangular R to matrix A
12)	R2RA	(R to R A)	Transfer a triangular block of R to triangular RA
13)	RUDR	(rudder)	SRIF R to U-D factors or vice versa
14)	THH	(T H H)	Triangular Householder data processing
15)	TRIMAT	(tri mat)	Triangular matrix print
16)	TTHH	(T T H H)	Two triangular matrix Householder processing
17)	TZERO	(T zero)	Zero a horizontal segment of a triangular matrix

18)	UDMES	(U D measurement)	U-D measurement updating
19)	UD2COV	(U D to cov)	U-D factors to covariance
20)	UD2SIG	(U D to sig)	U-D factors to sigmas
21)	UTINV	(U T inverse)	Upper triangular matrix inverse
22)	UTIROW		Upper triangular inverse, inverting only the upper rows
23)	WGS	(W G-S)	Weighted Gram-Schmidt triangular reduction

These routines are described in succeedingly more detail in sections III, IV, and V. The examples to follow are chosen to demonstrate how these various subroutines can be used to solve orbit determination and other parameter estimation problems. It is important to keep in mind that these examples are not by any means all inclusive, and that this package of subroutines has a wide scope of applicability.

II.1 Simple Least Squares

Given data in the form of an overdetermined systems of linear equations one may want a) the least squares solution; b) the estimate error covariance, assuming that the data has normalized errors; and c) the sum of squares of the residuals. The solution to this problem, using the ESP can be symbolically depicted as

- $[A\ z] \xrightarrow{THH} [\hat{R}\ \hat{z}],\ e$

Remarks: The array [A z] corresponds to the equations $Ax = z-\nu$, $\nu \epsilon N(0,I)$; the array $[\hat{R}\ \hat{z}]$ corresponds to the triangular data equation $\hat{R}x = \hat{z}-\hat{\nu}$, $\nu \epsilon N(0,I)$ and $e = ||z-A\hat{x}||$

- $[\hat{R}\ \hat{z}] \xrightarrow{UTINV} [\hat{R}^{-1}\ \hat{x}]$

Remark: $\hat{x} = \hat{R}^{-1}\ \hat{z}$

One may be concerned with the integrity of the computed inverse and the estimate. If one uses subroutine RINCON instead of UTINV then in addition one obtains an estimate (lower and upper bounds) for the condition number R. If this condition number estimate is large the computed inverse and estimate are to be regarded with suspicion. By large, we mean considerable with the machine accuracy (viz. on an 18 decimal digit machine numbers larger than 10^{15}). Note that the condition number estimate is obtained with negligible additional computation and storage.

- $[\hat{R}^{-1}] \xrightarrow{\text{RI2COV}} [C]$

Remarks: $C = \hat{R}^{-1}\,\hat{R}^{-T}$ = estimate error covariance. Some computation can be avoided in RI2COV if only some (or all) of the standard deviations are wanted.

II.2 Least Squares With A Priori

If a priori information is given, it can be included as additional equations (in the A array) or used to initialize the R array in subroutine THH (see the subroutine argument description given in section IV). One is sometimes interested in seeing how the estimate and/or the formal statistics change corresponding to the use of different a priori conditions. In this case one should compute $[\hat{R}\ \hat{z}]$ as in case II.1, and then include the a priori $[R_o\ z_o]$ using either subroutine THH, or subroutine TTHH when the a priori is diagonal or triangular, e.g.,

- $\left.\begin{matrix}[\hat{R}\ \hat{z}] \\ [R_o z_o]\end{matrix}\right\} \xrightarrow{\text{TTHH}} [\hat{R}\ \hat{z}]^*$

*The new result overwrites the old.

It is often good practice to process the data and form $[\hat{R}\ \hat{z}]$ before including the effects of a priori. When this is done one can analyze the effect of different a priori, $[R_o\ z_o]$ without reprocessing the data.

If a priori is given in the form of an information matrix, A, (as for example would be the case if the problem is being initialized with data processed using normal equation data accumulation*) then one can obtain R_o from A using INF2R;

$$A \xrightarrow{\text{INF2R}} R_o$$

If there were a normal equation estimate $z = A^T b$, then $z_o = R_o^{-T} z$.

II.3 Batch Sequential Data Processing

Prime reasons for batch sequential data processing are that many problems are too large to fit in core, are too expensive in terms of core cost, and for certain problems it is desirable to be able to incorporate new data as it becomes available. Subroutines TTH and UDMES are specially designed for this kind of problem. Both of these subroutines overwrite the a priori with the result which then acts as a priori for the next batch of data. If the data is stored on a file or tape as A_1, z_1, A_2, z_2,... then the sequential process can be represented as follows:

SRIF Processing**

a) Initialize [R z] with a-priori values or zero

b) Read the next [A z] from the file

---

*i.e., solving $Ax = b-v$ with normal equations, $A^T A\hat{x}_o = A^T b$; $A = A^T A$ is the information matrix.

**The acronym SRIF represents Square Root Information Filter. The SRIF is discussed at length in reference [3].

c) $\left.\begin{matrix}[\hat{R}\ \hat{z}] \\ [A\ z]\end{matrix}\right\} \xrightarrow{\text{THH}} [\hat{R}\ \hat{z}]^{*}$

d) If there is more data go back to b)

e) Compute estimates and/or covariances using UTINV and RI2COV (as in example II.1)

U-D** Processing

a′) Initialize $[\hat{U\text{-}D}\ \hat{x}]$ with a priori U-D information and estimate

b′) Read the next [A z] scalar measurement from the file

c′) $\left.\begin{matrix}[\hat{U\text{-}D}\ \hat{x}] \\ [A\ z]\end{matrix}\right\} \xrightarrow{\text{UDMES}} [\hat{U\text{-}D}\ \hat{x}]^{*}$

d′) If there is more data go back to b′)

e′) Compute standard deviations or covariances using UD2SIG or UD2COV.

Note that subroutine THH is best (most efficiently) used with data batches of substantial size (say 5 or more) and that UDMES processes measurement vectors one component at a time. If the dimension of the state is small the cost of using either method is generally negligible. The UDMES subroutine is best used in problems where estimates are wanted with great frequency or where one wishes to monitor the effects of each update. In a given application one might choose to process data in batches for awhile and during critical periods it may be

*The new result overwrites the old.

**U-D processing is a numerically stable algorithmic formulation of the Kalman filter measurement update algorithm, cf reference [3]. The estimate error covariance is used in its UDU^{T} factored form, where U is unit upper triangular and D is diagonal.

desirable to monitor the updating process on a point by point basis. In cases such as this, one may use RUDR to convert a SRIF array to U-D form or vice-versa.

Remarks: Another case where an R to U-D conversion can be useful occurs in large order problems (with say 100 or more parameters) where after data has been SRIF processed one wants to examine estimate and/or covariance sensitivity to the a priori variances of only a few of the variables. Here it may be more convenient to update using the UDMES subroutine.

II.4 Reduced State Estimates and/or Covariances From a SRIF Array

Suppose, for example, that data has been processed and that we have a triangular SRIF array $[\hat{R}\ \hat{z}]$ corresponding to the 14 parameter names, a_r, a_x, a_y, x, y, z, v_x, v_y, v_z, GM, CU41, LO41, CU43, LO43 (constant spacecraft accelerations, position and velocity, target body gravitational constant, and spin axis and longitude station location errors).

Let us ask first what would the computed error covariance be of a model containing only the first 10 variables, i.e., by ignoring the effect of the station location errors. One would apply UTINV and RI2COV just as in example II.1, except here we would use N (the dimension of the filter) = 10, instead of N=14.

Next, suppose that we want the solution and associated covariance of the model without the 3 acceleration errors. One ESP solution is to use

- $[\hat{R}\ \hat{z}] \xrightarrow{R2A} [A]$

NAME ORDER OF A

$x, y, z, v_x, v_y, v_z,$

GM, CU41, LO41, CU43, LO43,

RHS*, $a_r, a_x, a_y,$

Remark: One could also have used subroutine COMBO, with the desired namelist as simply a_r, a_x, a_y. This would achieve the same A matrix form.

- $[A] \xrightarrow{THH} [R]$

Remark: R here can replace the original $\hat{R}$ and $\hat{z}$.

- $[R] \xrightarrow{UTINV} [R^{-1}\ x_{est}] \xrightarrow{RI2COV} [COV\ x_{est}]$

Remarks: Here, use only N=11, i.e., 11 variables and the RHS. x_{est} is the 11 state estimate based on a model that does not contain acceleration errors a_r, a_x, or a_y.

Note how triangularizing the rearranged R matrix produces the desired lower dimensional SRIF array; and this is the same result one would obtain if the original data had been fit using the 11 state model.

As the last subcase of this example suppose that one is only interested in the SRIF array corresponding to the position and velocity variables. The difference between this example and the one above is that here we want to include the effects due to the other variables.

---

*z is often given the label RHS (right hand side)

One might want this sub-array to combine with a position-velocity SRIF array obtained from, say, optical data. One method to use would be,

- $[\hat{R}\ \hat{z}] \xrightarrow{R2RA} [R_A\ z_A]$

INPUT NAMES: OUTPUT NAMES:

a_r, a_x, a_y, x, y, z, v_x, v_y, v_z, GM x, y, z, v_x, v_y, v_z, GM

CU41, LO41, CU43, LO43, RHS CU41, LO41, CU43, LO43, RHS

Remark: The lower triangle starting with x is copied into R_A.

- $[R_A\ z_A] \xrightarrow{R2A} [A,\ z_A]$ (Reordering)

NAMES: GM, CU41, LO41, CU43, LO43,

x, y, z, v_x, v_y, v_z, RHS

- $[A,\ z_A] \xrightarrow{THH} [\hat{R}_A\ \hat{z}_A]$ (Triangularizing)
- $[\hat{R}_A\ \hat{z}_A] \xrightarrow{R2RA} [R_x\ z_x]$ (Shifting array)

NAMES: x, y, z, v_x, v_y, v_z, RHS

Remark: The lower right triangle starting with x is copied into R_x. We note that one could have elected to use COMBO in place of the first R2RA usage and R2A; this would have involved slightly more storage, but a lesser number of inputs. The sequence of operations is in this case,

- $[\hat{R}\ \hat{z}] \xrightarrow{COMBO} [A\ z]$

ORIGINAL NAMES DESIRED NAMES: x, y, z, v_x, v_y, v_z, RHS

Remark: By using COMBO the columns of $[\hat{R}\ \hat{z}]$ are ordered corresponding to the names a_r, a_x, a_y, GM, CU41, LO41, CU43, and LO43, followed by the desired names list.

- $[A\ z] \xrightarrow{THH} [\hat{R}\ \hat{z}]$

Remark: The $[\hat{R}\ \hat{z}]$ array that is output from this procedure is equivalent but different from the $[\hat{R}\ \hat{z}]$ array that we began with.

- $[\hat{R}\ \hat{z}] \xrightarrow{R2RA} [R_x\ z_x]$

Remark: As before, the lower right triangle starting with x is copied into R_x.

To delete the last k parameters from a SRIF array, it is not necessary to use subroutines R2A and THH. The first $N - k = \bar{N}$ columns of the array already correspond to a square root information matrix of the reduced system. If estimates are involved one can simply move the z column left using:

$$R(\bar{N}*(\bar{N}+1)/2 + i) = R(N*(N+1)/2 + i),\ i = 1,\ldots,k.$$

Remark: We mention in passing that if one is only interested in estimates and/or covariances corresponding to the last k parameters then one can use R2RA to transform the lower right triangle of the SRIF array to an upper left triangle after which UTINV and RI2COV can be applied.

II.5 Sensitivity, Perturbation, Computed Covariance and Consider Covariance Matrix Computation

Suppose that one is given a SRIF array

$$\begin{array}{ccc} \underbrace{}_{N_x} & \underbrace{}_{N_y} & \underbrace{}_{1} \\ \begin{bmatrix} R_x & R_{xy} & z_x \\ 0 & R_y & z_y \end{bmatrix} & \begin{matrix} \} N_x \\ \} N_y \end{matrix} \end{array} \qquad \text{(II.5a)}$$

in which the N_y variables are to be considered. (One can, of course, using subroutines R2A and THH reorder and retriangularize an arbitrarily arranged SRIF array so that a given set of variables fall at the end.) For various reasons one may choose to ignore the y variables in the equation

$$R_x x + R_{xy} y = z_x - \nu_x \,, \quad \nu_x \varepsilon N(0,I) \tag{II.5b}$$

and take as the estimate $x_c = R_x^{-1} z_x$. It then follows that

$$x - x_c = -R_x^{-1} R_{xy}\, y - R_x^{-1} \nu_x \,, \tag{II.5c}$$

and from this one obtains

$$\text{Sen} \equiv \frac{\partial(x-x_c)}{\partial y} = -R_x^{-1} R_{xy} \tag{II.5d}$$

(sensitivity of the estimate error to the unmodeled y parameters)

$$\text{Pert} = \text{Sen Diag}\,(\sigma_y(1),\ldots,\sigma_y(N_y)) \tag{II.5e}$$

where $\sigma_y(1),\ldots,\sigma_y(N_y)$ are a priori y parameter uncertainties.

(The perturbations are a measure of how much the estimate error could be expected to change due to the unmodeled y parameters.)

$$P_{con} = R_x^{-1} R_x^{-T} + \text{Sen}\, P_y\, \text{Sen}^T \tag{II.5f}$$

$$= P_c + (\text{Pert})(\text{Pert})^T \text{ if } P_y \text{ is diagonal*}$$

where P_c is the estimate error covariance of the reduced model.

An easy way to compute P_c, Pert and P_{con} is as follows: Use subroutine R2RA to place the y variable a priori $[P_y^{\frac{1}{2}}(0) \quad \hat{y}_o]$** into the lower right

*Pert = Sen $P_y^{\frac{1}{2}}$

**The a priori estimate y_o of consider parameters is generally zero.

corner of (II.5a), replacing R_y and z_y, i.e.,

$$\left.\begin{matrix}[\dot{R} & z] \\ [P_y^{\frac{1}{2}}(0) & \hat{y}_o]\end{matrix}\right\} \xrightarrow{R2RA} \begin{bmatrix} R_x & R_{xy} & z_x \\ 0 & P_y^{\frac{1}{2}}(0) & \hat{y}_o \end{bmatrix}$$

Now apply subroutine UTIROW to this system (with a -1 set in the lower right corner*)

$$\left[\begin{array}{ccc} R_x & R_{xy} & z_x \\ \hline 0 & P_y^{\frac{1}{2}}(0) & \hat{y}_o \\ 0 & 0 & -1 \end{array}\right] \xrightarrow{UTIROW} \left[\begin{array}{ccc} R_x^{-1} & \text{Pert}^{**} & x_c \\ \hline 0 & P_y^{\frac{1}{2}}(0) & \hat{y}_o \\ 0 & 0 & -1 \end{array}\right]$$

Note that the lower portion of the matrix is left unaltered, i.e., the purpose of UTIROW is to invert a triangular matrix, given that the lower rows have already been inverted. From this array one can, using subroutine RI2COV, get both P_c and P_{con}

$$[R_x^{-1}] \xrightarrow{RI2COV} [P_c] \qquad \text{computed covariance}$$

$$[R_x^{-1} \quad \text{Pert}] \xrightarrow{RI2COV} [P_{con}] \qquad \text{consider covariance}$$

Suppose now that one is dealing with a U-D factored Kalman filter formulation. In this case estimate error sensitivities can be sequentially

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* To have estimates from the triangular inversion routines one sets a -1 in the last column (below the right hand side).

** Strictly speaking this is not what we call the perturbation unless $R_y(0)$ is diagonal.

calculated as each scalar measurement ($z = a_x^T x + a_y^T y + \nu$) is processed.

$$Sen_j = Sen_{j-1} - K_j(a_x^T Sen_{j-1} + a_y^T)$$

where Sen_{j-1} is the sensitivity prior to processing this (j-th) measurement, and K_j is the Kalman gain vector. In this formulation one computes P_{con} in a manner analogous to that described in section II.7;

Let $\bar{U}_1 = U_j$, $\bar{D}_1 = D_j$ (filter U-D factors)

$[s_1, \ldots, s_{n_y}] = Sen_j$ (estimate error sensitivities)

then compute

$$\bar{U}_k\text{-}\bar{D}_k \text{ , } \sigma_k^2, s_k \xrightarrow{\text{AGTRN}} \bar{U}_{k+1}\text{-}\bar{D}_{k+1} \qquad k = 1, \ldots, n_y$$

For the final $\bar{U}$-$\bar{D}$ we have

$$U_{j+1}^{con} = \bar{U}_{n_y+1} \text{ , } D_{j+1}^{con} = D_{n_y+1}$$

If $P_y(0) = U_y D_y U_y^T$, instead of $P_y(0) = \text{Diag}(\sigma_1^2, \ldots, \sigma_{n_y}^2)$, then in the U-D recursion one should replace the Sen_j columns by those of $Sen_j U_j$ and σ_j^2 should be replaced by the corresponding diagonal elements of D_y.

II.6 Combining Various Data Sets

In this example we collect several related problems involving data sets with different parameter lists.

Suppose that the parameter namelist of the current data does not correspond to that of the a priori SRIF array. If the new data involves a permutation or a subset of the SRIF namelist then an application of

subroutine PERMUT will create the desired data rearrangement. If the data involves parameters not present in the SRIF namelist then one could use subroutine R2A to modify the SRIF array to include the new names and then if necessary use PERMUT on the data, to rearrange it compatibly.

Suppose now that two data sets are to be combined and that each contains parameters peculiar to it (and of course there are common parameters). For example let data set 1 contain names ABC and data set 2 contain names DEB. One could handle such a problem by noting that the list ABCDE contains both name lists. Thus one could use subroutine PERMUT on each data set comparing it to the master list, ABCDE, and then the results could be combined using subroutine THH An alternative automated method for handling this problem is to use subroutine COMBO with data set 1 (assuming it is in triangular form) and namelist 2. The result would be data set 1 in double subscripted form and arranged to the namelist ACDEB (names A and C are peculiar to data set 1 and are put first). Having determined the namelist one could apply subroutine PERMUT to data set 2 and give it a compatible namelist ordering.

The process of increasing the namelist size to accommodate new variables can lead to problems with excessively long namelists, i.e., with high dimension. If it is known that a certain set of variables will not occur in future data sets then these variables can be eliminated and the problem dimension reduced. To eliminate a vector y from a SRIF array, first use subroutine R2A to put the y names first in the namelist; then use subroutine THH to retriangularize and finally use subroutine R2RA to put the y independent subarray in position for further use; viz.

$$[R] \xrightarrow{R2A} [A] \xrightarrow{THH} \begin{bmatrix} R_y & R_{yx} & \bar{z}_y \\ 0 & R_x & z_x \end{bmatrix} \xrightarrow{R2RA} [R_x \; z_x]$$

The rows $[R_y \; R_{yx} \; z_y]$ can be used to recover a y estimate (and its covariance) when an estimate for x (and its covariance) are determined. (See example II.4).

Still another application related to the combining of data sets involves the combining of SRIF triangular data arrays. One might encounter such problems when combining data from different space missions (that involve common parameters) or one might choose to process data of each type* or tracking station separately and then combine the resulting SRIF arrays. Triangular arrays can be combined using subroutine TTHH, assuming that subroutines R2A, THH and R2RA have been used previously to formulate a common parameter set for each of the sub problems.

II.7 Batch Sequential White Noise

It is not uncommon to have a problem where each data set contains a set of parameters that apply only to that set and not to any other, viz. the data is of the form

$$A_j x + B_j y_j = z_j - \nu_j \qquad j = 1,\ldots,N$$

where there is generally a priori information on the vector y_j variables. Rather than form a concatenated state vector composed of x, $y_1,\ldots,y_N$ which might create a problem involving exhorbitant amounts of storage and computation we solve the problem as follows. Apply subroutine THH to $[B_1 \; A_1 \; z_1]$, with the corresponding R initialized with the y_1 a priori. The resulting SRIF array is of the form

*viz. range, doppler, optical, etc.

$$N_{y_1}\left\{\begin{bmatrix} R_{y_1} & R_{y_1 x} & z_{y_1} \\ 0 & R_{x_1} & z_{x_1} \end{bmatrix}\right.$$

Copy the top N_{y_1} rows if one will later want an estimate or covariance of the y_1 parameters. Apply subroutine TZERO to zero the top N_{y_1} rows and using subroutine R2RA set in the y_2 a priori*. This SRIF array is now ready to be combined with the second set of data $[B_2\ A_2\ Z_2]$ and the procedure repeated.

A somewhat analogous situation is represented by the class of problems that involve noisy model variations, i.e., the state at step j+1 satisfies

$$x_{j+1} = x_j + G_j w_j$$

where matrix G_j is defined so that w_j is independent of x_j and $w_j \epsilon N(0,Q_j)$. Models of this type are used to reflect that the problem at hand is not truly one of parameter estimation, and that some (or all) of the components vary in a random (or at least unknown) manner that is statistically bounded. To solve this problem in a SRIF formulation suppose that a priori for x_j and w_j are written in data equation form (cf ref. [3]),

$$R_j x_j = z_j - \nu_j \quad ; \quad \nu_j \epsilon N(0, I_{n_w})$$

$$Q_j^{-\frac{1}{2}} w_j = 0 - \nu_j^{(w)} \quad ; \quad \nu_j^{(w)} \epsilon N(0,I)$$

where $Q_j^{\frac{1}{2}}$ is a Cholesky factor of Q_j that is obtainable from COV2RI. Combining these two equations with the one for x_{j+1} gives

*In this example it is assumed that all of the y_j variables have the same dimension. This assumption, though not essential, simplifies our description of the procedure.

$$\begin{bmatrix} I_{n_w} & 0 \\ -R_j G_j Q_j^{\frac{1}{2}} & R_j \end{bmatrix} \begin{bmatrix} \hat{w}_j \\ x_{j+1} \end{bmatrix} = \begin{bmatrix} 0 \\ z_j \end{bmatrix} - \begin{bmatrix} \nu_j^{(w)} \\ \nu_j \end{bmatrix}$$

where $Q_j^{\frac{1}{2}} w_j = w_j$. This is the equation to be triangularized with subroutine THH, i.e.,

$$\begin{array}{l} \\ \text{Dim w} \\ \text{Dim x} \end{array} \begin{array}{c} \overbrace{\phantom{I_{n_w}}}^{\text{Dim w}} \quad \overbrace{}^{\text{Dim x}} \quad \overbrace{}^{1} \\ \left\{ \begin{bmatrix} I_{n_w} & 0 & 0 \\ -R_j G_j Q_j^{\frac{1}{2}} & R_j & z_j \end{bmatrix} \right. \end{array} \xrightarrow{\text{THH}} \begin{bmatrix} R_j^{(w)} & R_j^{(wx)} & z_j^w \\ 0 & R_{j+1} & z_{j+1} \end{bmatrix}$$

If the problem is arranged so that Q_j is diagonal one can reduce storage and computation. The form of this algorithm is designed to allow the use of singular Q_j matrices.

When the a priori for x_j and Q_j are given in U-D factored form, one can obtain the U-D factors for x_{j+1} as follows:

Let $Q_j = U^{(q)} D^{(q)} (U^{(q)})^T$ (use COV2UD if necessary)

Set $\bar{G} = G_j U^{(q)} = [g_1, \ldots, g_{n_w}]$, $D^{(q)} = \text{Diag}(d_1, \ldots, d_{n_w})$

Apply subroutine AGTRN n_w times, with $\bar{U}_1 = \bar{U}_j$, $\bar{D}_1 = D_j$

$$\left.\begin{array}{l} (\bar{U}\text{-}\bar{D})_k \; ; \; d_k, \; g_k \xrightarrow{\text{AGTRN}} (\bar{U}\text{-}\bar{D})_{k+1} \\ \text{i.e.} \quad (\bar{U}_k \bar{D}_k \bar{U}_k^T + d_k g_k g_k^T = \bar{U}_{k+1} \bar{D}_{k+1} \bar{U}_{k+1}^T) \end{array}\right\} \quad k = 1, \ldots, n_w$$

Then $U_{j+1} = \bar{U}_{n_w}$, $D_{j+1} = \bar{D}_{n_w}$.

Certain filtering problems involve dynamic models of the form

$$x_{j+1} = \Phi_j \, x_j + G_j \, w_j$$

Given an estimate for x_j, $\hat{x}_j$, the predicted estimate for x_{j+1}, denoted $\tilde{x}_{j+1}$ is simply*

$$\tilde{x}_{j+1} = \Phi_j \, \hat{x}_j$$

The U-D factors of the estimate error corresponding to the estimate $\tilde{x}_{j+1}$ can be obtained using the weighted Gram-Schmidt triangularization subroutine

$$\left[\Phi_j \; U_j \,|\, \bar{G}\right], \; \text{Diag}\,(D_j, D^{(q)}) \xrightarrow{W\,G\,S} (\tilde{U}_{j+1} - \tilde{D}_{j+1})$$

II.8 Miscellaneous Uses of the Various ESP Subroutines

In certain parameter analyses we may want to reprocess a set of data suppressing different subsets of variables. In this case the original data should be left unaltered and subroutine A2A1 used to copy A into A_1, which then can be modified as dictated by the analysis.

Covariance analyses sometimes are initialized using a covariance matrix from a different problem (or a differed phase of the same problem). In such cases it may be necessary to permute, delete or insert rows and columns into the covariance matrix; and that can be achieved using subroutine C2C.

If a priori for the problem at hand is given as a covariance matrix then one can compute the corresponding SRIF or U-D initialization using

*In statistical notation that is commonly used, one writes

$$x(j+1|j) = \Phi_j \, x(j|j)$$

subroutines COV2RI or COV2UD. Of course, if the covariance is diagonal the appropriate R and U-D factors can be obtained more simply. To convert a priori given in the form of an information matrix to a corresponding SRIF matrix one applies subroutine INF2R. To display covariance results corresponding to the SRIF or U-D filter one can use subroutines UTINV, R12COV and UD2COV. The vector stored covariance results are displayed in a triangular format using subroutine TRIMAT.

Aside: After careful consideration it was decided that subroutines to multiply matrices would not be included in our ESP. Our reasons are that parameter estimation does not, in the main, involve matrix multiplication; and when such products occur they generally involve matrices with special structures (viz. rectangle x triangle, triangle x rectangle, diagonal x triangle, etc). To see that these computations are not lengthy or complicated we illustrate how to compute z = Rx where R is a triangular vector stored matrix and x is an N vector,

```
      II=0
      DO 2 I=1,N
      SUM=0.
      II=II+I                    @II=(I,I)
      IK=II
      DO 1 K=I,N
      SUM=SUM+R(IK)*x(K)         @IK=(I,K)
1     IK=IK+K
2     z(I)=SUM                   @z can overwrite x if desired.
```

Note that the II and IK incremental recursions are used to circumvent the N(N+1)/2 calculations of IK=K(K-1)/2+I.

A later more encyclopedic subroutine directory may include the various matrix products that occur in linear algebra applications.

End of Aside

III. SUBROUTINE DIRECTORY SUMMARY

1. AGTRN - (Agee-Turner)

Computes updated U-D factors corresponding to a rank 1 matrix modification; i.e., given U-D, a scalar c, and vector v, $\bar{U}$ and $\bar{D}$ are computed so that $U\ D\ U^T = U\ D\ U^T + c\ v\ v^T$. Both c and v are destroyed during the computation, and the resultant (vector stored) U-D array replaces the original one. Uses for this routine include (a) adding process noise effects to a U-D factored Kalman filter; (b) computing consider covariances (cf Section II.5); (c) computing "actual" covariance factors resulting from the use of suboptimal Kalman filter gains; and (d) adding measurements to a U-D factored information matrix.

2. A2A1 - (A to A1)

Reorders the columns of a rectangular matrix A, storing the result in matrix A1. Columns can be deleted and new columns added. Zero columns are inserted which correspond to new column name entries. Matrices A and A1 cannot share common storage.

Example III.1

	α	B	C
	1	5	9
	2	6	10
	3	7	11
-	4	8	12

A

A2A1 →

B	F	G	C	H
5	0	0	9	0
6	0	0	10	0
7	0	0	11	0
8	0	0	12	0

A1

The new namelist (BFGCH) contains F, G and H as new columns and deletes the column corresponding to name α.

Example III.2

Suppose one is given an observation data file with regression coefficients corresponding to a state vector with components say, x, y, z, v_x, v_y, v_z and station location errors. Suppose further, that the vector being estimated has components a_r^*, a_x^*, a_y^*, x, y, z, v_x, v_y, v_z, GM and station location errors. A2A1 can be used to reorder the matrix of regression coefficients to correspond to the state being estimated. Zero coefficients are set in place for the accelerations and GM which are not present in the original file.

3. COMBO - (combine R and A namelists)

The upper triangular vector stored matrix R has its columns permuted and is copied into matrix A. The names associated with R are to be combined with a second namelist.

The namelist for A is arranged so that R names not contained in the second list appear first (left most). These are then followed by the second list. Names in the second list that do not appear in the R namelist have columns of zeros associated with them.

Example III.3

NAM2 list

α	B	C	D
1	2	4	7
0	3	5	8
0	0	6	9
0	0	0	10

→

C	B	E	α	F	D
4	2	0	1	0	7
5	3	0	0	0	8
6	0	0	0	0	9
0	0	0	0	0	10

R-Vector stored A-Double subscripted

* in track and cross track accelerations

A principal application of this subroutine is to the problem of combining equation sets containing different variables, and automating the process of combining name lists.

4. COV2RI - (Covariance to R inverse)

An input positive semi-definite vector stored matrix P is replaced by its upper triangular vector stored Cholesky factor U, $P = UU^T$. The name RI is used because when the input covariance is positive definite, $U = R^{-1}$.

5. COV2UD - (Covariance to U-D factors)

An input positive semi-definite vector stored matrix P is replaced by its upper triangular vector stored U-D factors. $P = UDU^T$.

6. C2C - (C to C)

Reorders the rows and columns of a square (double subscripted) matrix C and stores the result back in C. Rows and columns of zeros are added when new column entries are added.

Example III.4

$$\begin{array}{c} \\ A \\ B \\ \Gamma \end{array}\begin{array}{c} \begin{array}{ccc} A & B & \Gamma \end{array} \\ \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \end{array} \xrightarrow{C2C} \begin{array}{c} \\ \Gamma \\ P \\ B \\ Q \end{array}\begin{array}{c} \begin{array}{cccc} \Gamma & P & B & Q \end{array} \\ \begin{bmatrix} 9 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Names P and Q have been added and name A deleted. An important application of this subroutine is to the rearranging of covariance matrices.

7. INF2R - (Information matrix to R)

Replaces a vector stored positive semi-definite information matrix Λ by its lower triangular Cholesky factor R^T; $\Lambda = R^T R$. The upper triangular matrix R is in the form utilized by the SRIF algorithms. The algorithm is designed to handle singular matrices because it is a

common practice to omit a priori information on parameters that are either poorly known or which will be well determined by the data.

8. PERMUT

Reorders the columns of matrix A, storing the result back in A. This routine differs from A2A1 principally in that here the result overwrites A. PERMUT is especially useful in applications where storage is at a premium or where the problem is of a recursive nature.

9. RINCON - (R inverse with condition number bound, CNB)

Computes the inverse of an upper triangular vector stored matrix R using subroutine UTINV. A Frobenius bound (CNB) for the condition number of R is computed too. This bound acts as both an upper and a lower bound, because CNB/N ≤ condition number ≤ CNB. When this bound is within several orders of magnitude of the machine accuracy the computed inverse is not to be trusted, (viz if CNB $\geq 10^{15}$ on an 18 decimal digit machine R is ill-conditioned).

10. RI2COV - (RI to covariance)

This subroutine computes sigmas (standard deviations) and/or the covariance of a vector stored upper triangular square root covariance matrix, RINV (SRIF inverse). The result, stored in COVOUT (covariance output) is also vector stored. COVOUT can overwrite RINV.

11. R2A - (R to A)

The columns of a vector stored upper triangular matrix R are permuted and variables are added and/or deleted. The result is stored in the double subscripted matrix A. In other respects the subroutine is like A2A1.

Example III.5

$$\begin{array}{c} \begin{matrix} \alpha & B & C & D & E \end{matrix} \\ \begin{bmatrix} 2 & 4 & 8 & 14 & 22 \\ 0 & 6 & 10 & 16 & 24 \\ 0 & 0 & 12 & 18 & 26 \\ 0 & 0 & 0 & 20 & 28 \\ 0 & 0 & 0 & 0 & 30 \end{bmatrix} \\ R \end{array} \xrightarrow{R2A} \begin{array}{c} \begin{matrix} E & F & C & B \end{matrix} \\ \begin{bmatrix} 22 & 0 & 8 & 4 \\ 24 & 0 & 10 & 6 \\ 26 & 0 & 12 & 0 \\ 28 & 0 & 0 & 0 \\ 30 & 0 & 0 & 0 \end{bmatrix} \\ A \end{array}$$

R is vector stored as R = (2,4,6,8,10,12,14,16,18,20,22,24,26,28,30) with namelist (α,B,C,D,E) associated with it. Names α and D are not included in matrix A, and a column of zeros corresponding to name F is added.

One trivial, but perhaps useful, application is to convert a vector stored matrix to a double subscripted form.* R2A is used most often when one wants to rearrange the columns of a SRIF array so that reduced order estimates, sensitivites, etc. can be obtained; or so that data sets containing different parameters can be combined.

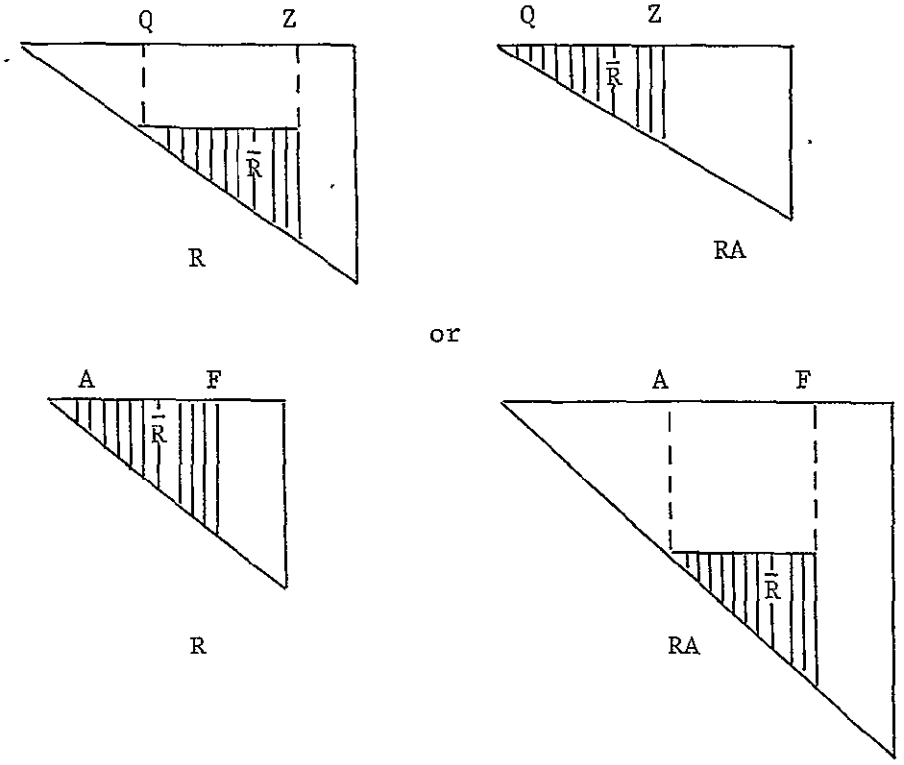
12. R2RA - (Triangular block of R to triangular block of RA)

A triangular portion of the vector stored upper triangular matrix R is put into a triangular portion of the vector stored matrix RA. The names corresponding to the relocated block are also moved. R can coincide with RA.

---

*see also the aside in the introduction

Examples III.6



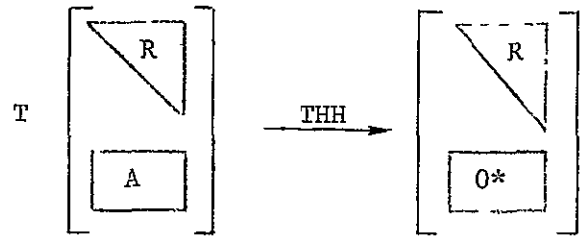
Note that an upper left triangular submatrix can slide to any lower position along the diagonal, but that a submatrix moving up must go to the upper leftmost corner. Upper shifting is used when one is interested in that subsystem; and the lower shifting is used, for example, when inserting a priori information for consider analyses.

13. RUDR - (SRIF R converted to U-D form or vice versa)

A vector stored SRIF array is replaced by a vector stored U-D form or conversely. A point to be noted is that when data is involved the right side of the SRIF data equation transforms to the estimate in the U-D array.

14. THH - (Triangular Householder data packing)

An upper triangular vector stored matrix R is combined with a rectangular doubly subscripted matrix A by means of Householder orthogonal transformations. The result overwrites R, and A is destroyed in the process.



15. TRIMAT - (Triangular matrix print)

Prints a vector stored upper triangular matrix, using a matrix format.

Example III.7

R(10) = (2,4,6,8,10,12,14,16,18,20) with associated namelist (A,B,C,D) is printed as

	A	B	C	D
A	2	4	8	14
B		6	10	16
C			12	18
D				20

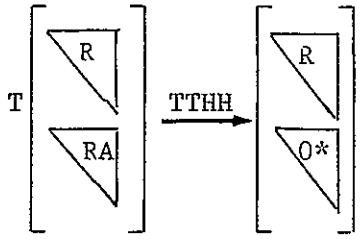
(The numbers are printed to 8 significant floating point digits).

To appreciate the importance of this subroutine compare the vector R(10) with the double subscript representation.

16. TTHH - (Two triangular arrays are combined using Householder orthogonal transformations)

This subroutine combines two single subscripted upper triangular SRIF arrays, R and RA using Householder orthogonal transformations. The result overwrites R.

*The elements are not explicitly set to zero.



17. TZERO - (Zero a horizontal segment of a vector stored upper triangular matrix)

Upper triangular vector stored matrix R has its rows between ISTART and IFINAL set to zero.

Example III.8

To zero row 2 and 3 of R(15), in the example of subroutine 11.

R(15) = (2,4,6,8,10,12,14,16,18,20,22,24,26,28,30)

R(15) = (2,4,0,8,0,0,14,0,0,20,22,0,0,28,30)

i.e.,

$$\begin{bmatrix} 2 & 4 & 8 & 14 & 22 \\ 0 & 6 & 10 & 16 & 24 \\ 0 & 0 & 12 & 18 & 26 \\ 0 & 0 & 0 & 20 & 28 \\ 0 & 0 & 0 & 0 & 30 \end{bmatrix} \xrightarrow{\text{TZERO}} \begin{bmatrix} 2 & 4 & 8 & 14 & 22 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 28 \\ 0 & 0 & 0 & 0 & 30 \end{bmatrix}$$

R-vector stored R-vector stored

*The elements are not explicitly set to zero.

18. UDMES - (U-D measurement update)

Given the U-D factors of the a priori estimate error covariance and the measurement, $z = Ax + \nu$ this routine computes the updated estimate and U-D covariance factors, the predicted residual, the predicted residual variance, and the normalized Kalman gain. This is Bierman's U-D measurement update algorithm.

19. UD2COV - (U-D factors to covariance)

The input vector stored U-D matrix (diagonal D elements are stored as the diagonal entries of U) is replaced by the covariance P, also vector stored. $P = UDU^T$. P can overwrite U to economize on storage.

20. UD2SIG - (U-D factors to sigmas)

Standard deviations corresponding to the diagonal elements of the covariance are computed from the U-D factors. This subroutine, a restricted version of UD2COV can print out the resulting sigmas and a title. The input U-D matrix is unaltered.

21. UTINV - (Upper triangular matrix inversion)

An upper triangular vector stored matrix RIN(R in) is inverted and the result, vector stored, is put in ROUT(R out). ROUT can overwrite RIN to economize on storage. If a right hand side is included and the bottommost tip of RIN has a -1 set in then ROUT will have the solution in the place of the right hand side.

22. UTIROW - (Upper triangular inversion, inverting only the upper rows)

$$\begin{matrix} & \text{INPUT} \\ n_y & \end{matrix} \begin{bmatrix} R_x & R_{xy} \\ \hline 0 & R_y^{-1} \end{bmatrix} \xrightarrow{\text{UTIROW}} \begin{bmatrix} R_x^{-1} & -R_x^{-1} R_{xy} R_y^{-1} \\ \hline 0 & R_y^{-1} \end{bmatrix} \quad \text{OUTPUT}$$

An input vector stored R matrix with its lower left triangle assumed to have been already inverted is used to construct the upper rows of the matrix inverse of the result. The result, vector stored, can overwrite the input to economize on storage.

If the columns comprising R_{xy} represent consider terms then taking R_y^{-1} as the identity gives the sensitivity on the upper right portion of the result. If $R_y^{-1} = \text{Diag}(\sigma_y, \ldots, \sigma_{n_y})$ then the upper right portion of the result represents the perturbation. Note that if z (the right hand side of the data equation) is included in R_{xy} then taking the corresponding R_y^{-1} diagonal as -1 results in the filter estimate appearing as the corresponding column of the output array. When n_y is zero this subroutine is equivalent to UTINV.

23. WGS - (Weighted Gram Schmidt matrix triangularization)

An input rectangular (possibly square) matrix W and a diagonal weight matrix, D_w, are transformed to (U-D) form; i.e.,

$$S\, D_w\, W^T = UDU^T$$

where U is unit upper triangular and D is diagonal. The weights D_w are assumed nonnegative, and this characteristic is inherited by the resulting D.

IV. SUBROUTINE DIRECTORY USER DESCRIPTION

1. AGTRN (Agee-Turner U-D rank one modification)

Purpose

To compute the (updated) U-D factors of $UDU^T + CVV^T$.

```
CALL AGTRN (UIN,UOUT,N,C,V)
```

Argument Definitions

UIN(N*(N+1)/2)	Input vector stored positive semi-definite U-D array (with the D entries stored on the diagonal of U)
UOUT(N*(N+1)/2)	Output vector stored result UOUT=UIN is allowed
N	Matrix dimension
C	Input scalar, destroyed by the algorithm
V(N)	Input vector, destroyed by the algorithm

Remarks and Restrictions

If C negative is used the algorithm is numerically unstable, and the result may be numerically unreliable. Singular U matrices are allowed, and these can result in singular output U matrices.

Functional Description

This rank one modification is based on a result published by Agee and Turner (1972), White Sands Missile Range Tech. Report No. 38. See also Ref. [3] where the algorithm is derived using geometric arguments.

2. A2A1 (A to A1)

Purpose

To rearrange the columns of a namelist indexed matrix to conform to a desired namelist.

```
CALL A2A1(A,IA,IR,LA,NAMA,A1,IA1,LA1,NAMA1)
```

Argument Definitions

A(IR,LA)	Input rectangular matrix
IA	Row dimension of A, IA.GE.IR
IR	Number of rows of A that are to be arranged
LA	Number of columns in A; this also represents the number of parameter names associated with A
NAMA(LA)	Parameter names associated with A
A1(IR,LA1)	Output rectangular matrix
IA1	Row dimension of A1, IA1.GE.IR
LA1	Number of columns in A1; this also represents the number of parameter names associated with A1
NAMA1(LA1)	Input list of parameter names to be associated with the output matrix A1

Remarks and Restrictions

A1 cannot overwrite A. This subroutine can be used to add on columns corresponding to new names and/or to delete variables from an array.

Functional Description

The columns of A are copied into A1 in an order corresponding to the NAMA1 parameter namelist. Columns of zeros are inserted in those A1 columns which do not correspond to names in the input parameter namelist NAMA.

3. COMBO (Combine parameter namelists)

Purpose

To rearrange a vector stored triangular matrix and store the result in matrix A. The difference between this subroutine and R2A is that there the namelist for A is input; here it is determined by combining the list for R with a list of desired names.

```
CALL COMBO  (R,L1,NAM1,L2,NAM2,A,IA,LA,NAMA)
```

Argument Definitions

R(L1*(L1+1)/2)	Input vector stored upper triangular matrix
L1	No. of parameters in R (and in NAM1)
NAM1(L1)	Names associated with R
L2	No. of parameters in NAM2
NAM2(L2)	Parameter names that are to be combined with R (NAM1 list); these names may or may not be in NAM1
A(L1,LA)	Output array containing the rearranged R matrix L1.LE.IA
IA	Row dimension of A
LA	No. of parameter names in NAMA, and the column dimension of A. LA = L1 + L2 - No. names common to NAM1 and NAM2; LA is computed and output
NAMA(LA)	Parameter names associated with the output A matrix ; consists of names in NAM1 not in NAM2 followed by NAM2

Remarks and Restrictions

The column dimension of A is a result of this subroutine. To avoid having A overwrite neighboring arrays one can bound the column dimension of A by L1 + L2.

Functional Description

First the NAM1 and NAM2 lists are compared and the names appearing in NAM1 only have their corresponding R column entries stored in A (e.g. if NAM1(2) and NAM1(6) are the only names not appearing in the NAM2 list then columns 2 and 6 of R are copied into columns 1 and 2 of A). The remaining columns of A are labeled with NAM2. The A namelist is recorded in NAMA. The NAM1 list is compared with NAM2 and matching names have their R column entries copied into the appropriate columns of A. NAM2 entries not appearing in NAM1 have columns of zero placed in A.

4. COV2RI (Covariance to Cholesky Square Root, RI)

Purpose

To construct the upper triangular Cholesky factors of a positive semi-definite matrix. Both the input covariance and the output Cholesky factor (square root) are vector stored. The output overwrites the input. Covariance (input) = U*U**T (output U = Rinverse).

```
CALL COV2RI(U,N)
```

Argument Definitions

U(N*(N+1)/2)	Contains the input vector stored covariance matrix (assumed positive definite)and on output it contains the upper triangular square root factor
N	Dimension of the matrices involved

Remarks and Restrictions

No check is made that the input matrix is positive semi-definite. Singular factors (with zero columns) are obtained if the input is (a) in fact singular, (b) ill-conditioned, or (c) in fact indefinite; and the latter two situations are cause for alarm. Case (c) and possibly (b) can be identified by using RI2COV to reconstruct the input matrix.

Functional Description

An upper triangular Cholesky reduction of the input matrix is implemented using a geometric algorithm described in Ref. [3].

$$U(\text{input}) = U(\text{output}) * U(\text{output})^{T}$$

At each step of the reduction diagonal testing is used and negative terms are set to zero.

5. COV2UD (Covariance to UD factors)

Purpose

To obtain the U-D factors of a positive semi-definite matrix. The input vector stored matrix is overwritten by the output U-D factors which are also vector stored.

```
CALL COV2UD(U,N)
```

Argument Definitions

U(N*(N+1)/2)	Contains the input vector stored covariance matrix; on output it contains the vector stored U-D covariance factors.
N	Matrix dimension

Remarks and Restrictions

No checks are made in this routine to test that the input U matrix is positive semi-definite. Singular results (with zero columns) are obtained if the input is (a) in fact singular, (b) ill-conditioned, or (c) in fact indefinite; and the latter two situations are cause for alarm. Case (c) and possibly case (b) can be identified by using UD2-COV to reconstruct the input matrix. Note that although indefinite matrices have U-D factorizations, the algorithm here applies only to matrices with non-negative eigenvalues.

Functional Description

An upper triangular U-D Cholesky factorization of the input matrix is implemented using a geometric algorithm described in Ref. [3].

$$U(\text{input}) = U*D*U^T, \qquad \text{U-D stored in U on output}$$

at each step of the reduction diagonal testing is used to zero negative terms.

6. C2C (C to C)

Purpose

To rearrange the rows and columns of C, from NAM1 order to NAM2 order. Zero rows and columns are associated with output defined names that are not contained in NAM1.

```
CALL C2C(C,IC,L1,NAM1,L2,NAM2)
```

Argument Definitions

C(L1,L1)	Input matrix
IC	Row dimension of C IC.GE.L = MAX(L1,L2)
L1	No. of parameter names associated with the input C
NAM1(L)	Parameter names associated with C on input. (Only the first L1 entries apply to the input C)
L2	No. of parameter names associated with the output C
NAM2(L2)	Parameter names associated with the output C

Remarks and Restrictions

The NAM2 list need not contain all the original NAM1 names and L1 can be .GE. or .LE. L2. The NAM1 list is used for scratch and appears permuted on output. If L2.GT.L1 the user must be sure that NAM1 has L2 entries available for scratch purposes.

Functional Description

The rows and columns of C and NAM1 are permuted pairwise to get the names common to NAM1 and NAM2 to coalesce. Then the remaining rows and columns of C(L2,L2) are set to zero.

7. INF2R (Information matrix to R)

Purpose

To compute a lower triangular Cholesky factorization of the input positive semi-definite matrix. The result transposed, is vector stored; this is the form of an upper triangular SRIF matrix.

```
CALL INF2R(P,N)
```

Argument Definitions

P(N*(N+1)/2)	Input vector stored positive semi-definite (information) matrix; on output it represents the transposed lower triangular Cholesky factor (i.e. the SRIF R matrix)
N	Matrix dimension

Remarks and Restrictions

No checks are made on the input matrix to guard against negative eigenvalues of the input, or to detect ill-conditioning. Singular output matrices have one or more rows of zeros.

Functional Description

A Cholesky type lower triangular factorization of the input matrix is implemented using the geometric formulation described in Ref. [3].

$$U(\text{input}) = [U(\text{output})]^T * [U(\text{output})]$$

At each step of the factorization diagonal testing is used to zero columns corresponding to negative entries. The result is vector stored in the form of a square root information matrix as it would be used for SRIF analyses.

8. PERMUT (Permute A)

Purpose

To rearrange the columns of a namelist indexed matrix to conform to a desired namelist. The resulting matrix is to overwrite the input.

```
CALL PERMUT(A,IA,IR,L1,NAM1,L2,NAM2)
```

Argument Definitions

A(IR,L)	Input rectangular matrix, L = max(L1,L2)
IA	Row dimension of A, IA.GE.IR
IR	Number of rows of A that are to be rearranged
L1	Number of parameter names associated with the input A matrix
NAM1(L)	Parameter names associated with A on input (only the first L1 entries apply to the input A)
L2	Number of parameter names associated with the output A matrix
NAM2	Parameter names associated with the output A

Remarks and Restrictions

This subroutine is similar to A2A1; but because the output matrix in this case overwrites the input there are several differences. The NAM1 vector is used for scratch, and on output it contains a permutation of the input NAM1 list. The user must allocate L = max(L1,L2) elements of storage to NAM1. The extra entries, when L2 > L1, are used for scratch.

Functional Description

The columns of A are rearranged, a pair at a time, to match the NAM2 parameter namelist. The NAM1 entries are permuted along with the columns, and this is why dim (NAM1) must be larger than L1 (when L2>L1). Columns of zeroes are inserted in A which correspond to output names that do not appear in NAM1.

9. RINCON (R inverse with condition number bound)

Purpose

To compute the inverse of an upper triangular vector stored triangular matrix, and an estimate of its condition number.

```
CALL RINCON(RIN,N,ROUT,CNB)
```

Argument Definitions

RIN(N*(N+1)/2)	Input vector stored upper triangular matrix
N	Matrix dimension
ROUT(N*(N+1)/2)	Output vector stored matrix inverse (RIN = ROUT is permitted)
CNB	Condition number bound. If κ is the condition number of RIN, then CNB/N.LE.κ.LE CNB

Remarks and Restrictions

The condition number bound, CNB serves as an estimate of the actual condition number. When it is large the problem is ill-conditioned. The matrix inversion is computed using subroutine UTINV.

Functional Description

The matrix inversion, a triangular back substitution, is accomplished via subroutine UTINV. If any diagonal element of the input R matrix is zero the inversion is not attempted; instead a message is printed. The condition number bound is computed as follows:

$$\text{F.NORM } R = \sum_{J=1}^{NTOT} R(J)^2$$

$$\text{F.NORM } R^{-1} = \sum_{J=1}^{NTOT} R^{-1}(J)^2$$

where NTOT = N*(N+1)/2 is the number of elements in the vector stored triangular matrix. The condition number bound, CNB, is given by

$$CNB = (\text{F.NORM } R * \text{F.NORM } R^{-1})^{1/2}$$

F.NORM is the Frobenius norm, squared. The inequality

$$CNB/N \leq \text{condition number } R \leq CNB$$

is a simple consequence of the Frobenius norm inequalities given in Lawson-Hanson "Solving Least Squares," page 234.

10. RI2COV (RI Triangular to covariance)

Purpose

To compute the covariance matrix and/or the standard deviation of a vector stored upper triangular square root covariance matrix. The output covariance matrix, also vector stored, may overwrite the input.

```
CALL RI2COV(RINV,N,SIG,COVOUT,KOV)
```

Argument Definitions

RINV(N*(N+1)/2)		Input vector stored upper triangular covariance square root (RINV=R inverse is the inverse of the SRIF matrix).
N		Dimension of the RINV matrix
SIG(N)		Output vector of standard deviations
COVOUT(N*(N+1)/2)		Output vector stored covariance matrix (COVOUT = RINV is allowed)
KOV	.GT.0	Compute covariance and sigmas using the first KOV rows of RINV
	.LT.0	Compute only the sigmas using the first KOV rows of RINV
	.EQ.0	No covariance, but all sigmas (e.g. use all N rows of RINV)

Remarks and Restrictions

Replacing N by |KOV| corresponds to computing the covariance of a lower dimensional system.

Functional Description

COVOUT=RINV*RINV**T.

11. R2A (R to A)

Purpose

To place the upper triangular vector stored matrix R into the matrix A and to arrange the columns to match the desired NAMA parameter list. Names in the NAMA list that do not correspond to any name in NAMR have zero entries in the corresponding A columns.

```
CALL R2A(R,LR,NAMR,A,IA,LA,NAMA)
```

Argument Definitions

R(LR*(LR+1)/2)	Input upper triangular vector stored array
LR	Row dimension of vector stored R
NAMR(LR)	Parameter names associated with R
A(LR,LA)	Matrix to house the rearranged R matrix
IA	Row dimension of A, IA.GE.LR.
LA	No. of parameter names associated with the output A matrix.
NAMA(LA)	Parameter names for the output A matrix.

Functional Description

The matrix A is set to zero and then the columns of R are copied into A.

12. R2RA (Permute a subportion R_A of a vector stored triangular matrix)

Purpose

To copy the upper left (lower right) portion of a vector stored upper triangular matrix R into the lower right (upper left) portion of a vector stored triangular matrix RA.

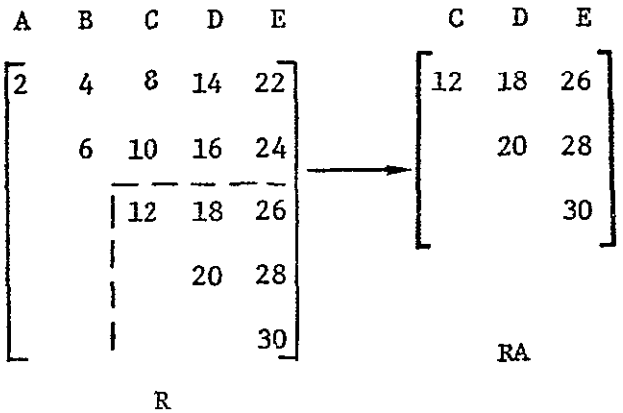
```
CALL R2RA(R,NR,NAM,RA,NRA,NAMA)
```

Argument Definitions

R(NR*(NR+1)/2)	Input vector stored upper triangular matrix
NR	Dimension of vector stored R matrix†
NAM(NR)	Names associated with R.
RA(NRA*(NRA+1)/2)	Output vector stored upper triangular matrix
NRA	If NRA = 0 on input, then NAMA(1) should have the first name of the output namelist. In this case the number of names in NAMA, NRA, will be computed. The lower right block of R will be the upper left block of RA. If NRA = last name of the upper left block that is to be moved then this upper block is to be moved to the lower right corner of RA. When used in this mode NRA=NR on output†.
NAMA(NRA)	Names associated with RA. Note that NRA used here denotes the output value of NRA.

Remarks and Restrictions

RA and NAMA can overwrite R and NAM. The meaning of the NRA = 0 option is clarified by the following example:



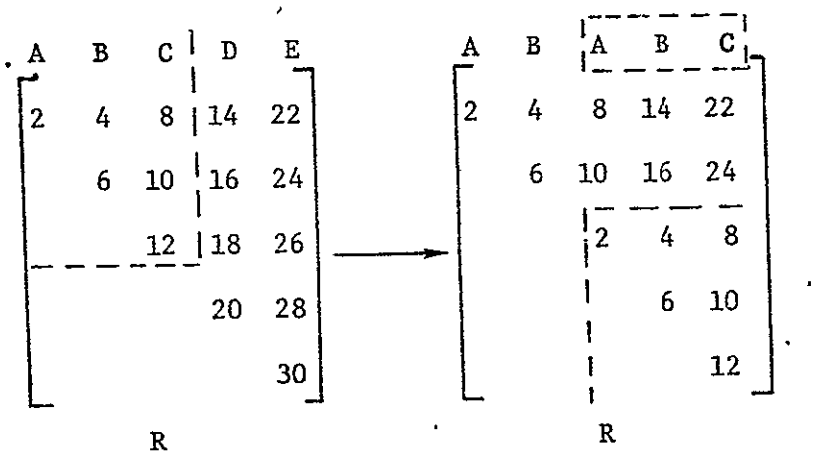
INPUT
NR = 5
NAM = 'A','B','C','D','E'
NRA = 0
NAMA(1) = 'C'
R

OUTPUT
NAMA = 'C', 'D', 'E'

†see the concluding paragraph of Remarks and Restrictions

When NRA = 0 and NAMA(1) = 'C' we are asking that the lower triangular portion of R, beginning at the column labeled C, be moved to form the first (in this case 3) columns of RA. Incidentally, RA could have additional columns; these columns and their names would be unaltered by the subroutine.

The meaning of the other NRA option is illustrated by the following example;



INPUT
NR = 5
NAM = 'A','B','C','D','E'
NRA = 'C'
R

OUTPUT
NRA = 5
NAMA(3-5) = 'A','B','C'
RA

When NRA = 'C' we are asking that the upper left block of R, up to the column labeled C, be moved to the lower left portion of RA and the corresponding names be moved too. If RA overwrites R, as in the example, then the first two rows of R remain unchanged and since NAMA overwrites NAM, the labels of the first two columns remain unaltered.

The remark that NRA=NR on output means, in this example, that the column with name C in R is moved over to column 5. If one wanted to slide the upper left triangle corresponding to names ABC of R to columns 7-9 of an RA matrix (of unspecified dimension, ≥ 9), then one should set NR=9 in the subroutine call. Thus NR, when used in this sliding down the diagonal mode, does not represent the dimension of R; but indicates how far the slide will be.

13. RUDR (R to U-D or U-D to R)

Purpose

To transform an upper triangular vector stored SRIF array to U-D form or vice versa.

```
CALL RUDR(RIN,N,ROUT,IS)
```

Argument Definitions

RIN(NBAR*(NBAR+1)/2)	Input upper triangular vector stored SRIF or U-D array; NBAR = ABS(N) + 1
ROUT(NBAR*(NBAR+1)/2)	Output upper triangular vector stored U-D or SRIF array (RIN = ROUT is permitted)
N	Matrix dimension, N.GT.0 represents an R to U-D conversion and N.LT.0 represents a U-D to R conversion.
IS	If IS = 0 the input array is assumed not to contain a right side (or an estimate), and IS = 1 means an appropriate additional column is included. In the IS = 0 case the last column of RIN is ignored and NBAR = ABS(N) is used.

Subroutine used: UTINV

Functional Description

Consider the N>0 case. RIN = R is transformed to ROUT = R inverse using subroutine UTINV with dimension N+IS. If IS = 1 the subroutine sets RIN((N+1)(N+2))/2) = -1. so that the N+1st column of ROUT will be the X estimate followed by -1. $R^{-1} = UD^{1/2}$ so that the diagonals are square root scaled U columns. This information is used to construct the U-D array which overwrites ROUT.

If N<0 the input is assumed to be a U-D array. This array is converted to ROUT=$UD^{\frac{1}{2}}$ and then using UTINV, R is computed and stored in ROUT. If IS = 1 the U-D matrix is assumed augmented by X (estimate), and on output the right side term of the SRIF array is obtained.

14. THH (Triangular Householder Orthogonalization)

Purpose

To compute [R z] such that

$$T \begin{bmatrix} \tilde{R} & \tilde{z} \\ A & z \end{bmatrix} = \begin{bmatrix} \hat{R} & \hat{z} \\ 0 & e \end{bmatrix} \quad T \text{ - orthogonal}$$

This is the key algorithm used in the square root information batch sequential filter.

```
CALL THH(R,N,A,IA,M,SOS,NSTRT)
```

Argument Definitions

R(N*(N+3)/2)	Input upper triangular vector stored square root information matrix. If estimates are involved SOS.GE.0 and R is augmented with the right hand side (stored in the last N locations of R). If SOS.LT.0 only the first N*(N+1)/2 locations of R are used. The result of the subroutine overwrites the input R
N	No. of parameters
A(M,N+1)	Input measurement matrix. The N+1st column is only used if SOS.GE.0, in which case it represents the right side of the equation $\nu + AX = z$. A is destroyed by the algorithm, but it is not explicitly set to zero.
IA	Row dimension of A
M	The number of rows of A that are to be combined with R
SOS	Accumulated residual sum of squares corresponding to the data processed prior to this time. On exit SOS represents the updated sum of squares of the residuals $\sum_i \|\|z_i - A_i X_{est}\|\|^2$, summed over the old and new data. It also includes the a priori term $\|\|R_o X_{est} - z_o\|\|^2$. Because SOS cannot be used if data, z, is not included we use SOS.LT.0 to indicate when data is

not included.

NSTRT	First column of the input A matrix that has a nonzero entry. In certain problems, especially those involving the inclusion of a priori statistics, it is known that the first NSTRT-1 columns of A all have zero entries. This knowledge can be used to reduce computation. If nothing is known about A then NSTRT.LE.1 gives a default value of 1, i.e. it is assumed that A may have nonzero entries in the very first column.

Remarks and Restrictions

It is trivial to arrange the code so that R output need not overwrite the input R. This was not done because, in the author's opinion, there are too few times when one desires to have ROUT ≠ RIN.

Functional Description

Assume for simplicity that NSTRT = 1. Then at step j, j = 1,...,N (or N+1 if data is present) the algorithm implicitly determines an elementary Householder orthogonal transformation which updates row j of R and all the columns of A to the right of the jth. At the completion of this step column j of A is in theory zero, but it is not explicitly set to zero. The orthogonalization process is discussed at length in the books by Lawson and Hanson, [1] and Bierman [3].

15. TRIMAT (Triangular matrix print)

Purpose

To display a vector stored upper triangular matrix in a two dimensional 8-digit triangular format.

```
CALL TRIMAT(A,N,CAR,TEXT,NCHAR,NAMES)
```

Argument Definitions

A(N*N+1)/2)	Vector stored upper triangular matrix
N	Dimension of A
CAR(N)	Parameter names (alphanumeric) associated with A
TEXT(NCHAR)	An array of field data characters to be printed as a title preceding the matrix
NCHAR	No. of characters (including spaces) that are to be printed in text( ) ABS(NCHAR).LE.126.NCHAR negative is used to avoid skipping to a new page to start printing
NAMES	A logical flag. If NAMES=.F. the CAR namelist is ignored and the columns and rows of A on output appear with numerical column heads

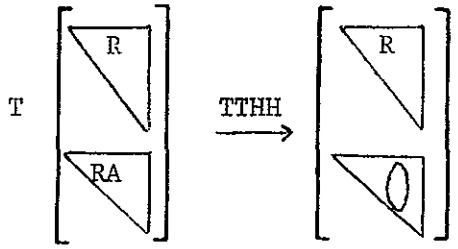
Remarks and Restrictions

Using NCHAR nonnegative, and starting the print at the top of a new page makes it easier to locate the printed result and is especially recommended when dealing with large dimensioned arrays. Page economy can, however, be achieved using the NCHAR negative option. In this case the print begins on the next line.

16. TTHH (Two triangular matrix Householder reduction)

Purpose

To combine two vector stored upper triangular matrices, R and RA by applying Householder orthogonal transformations. The result overwrites R.



CALL TTHH (R,RA,N)

Argument Definitions

R(N*(N+1)/2)	Input vector stored upper triangular matrix, which also houses the result
RA(N*(N+1)/2)	Second input vector stored upper triangular matrix. This matrix is destroyed by the computation.
N	Matrix dimension N less than zero is used to indicate that R and RA have right sides (|N|+1 columns) and have dimension |N|*(|N|+3)/2).

Remarks and Restrictions

RA is theoretically zero on output, but is not set to zero.

17. TZERO (Triangular matrix zero)

Purpose

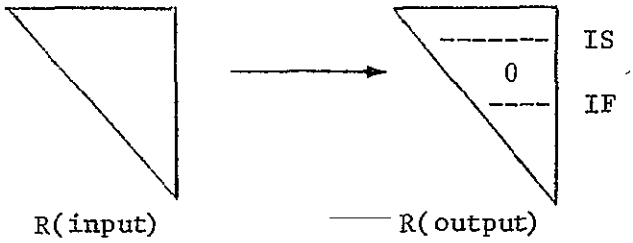
To zero out rows IS(Istart) to IF(Ifinal) of the vector stored upper triangular matrix R.

```
CALL TZERO(R,N,IS,IF)
```

Argument Definition

R(N*(N+1)/2)	Input vector stored upper triangular matrix
N	Row dimension of vector stored matrix
IS	First row of R that is to be set to zero
IF	Last row of R that is to be set to zero

Functional Description



18. UDMES (U-D measurement update)

Purpose

Kalman filter measurement updating using Bierman's U=D measurement update algorithm, cf 1975 CONF. DEC. CONTROL paper. A scalar measurement $z = A^T x + \nu$ is processed, the covariance U-D factors and estimate (if included) are updated, and the Kalman gain and innovations variance are computed.

```
CALL UDMES(U,N,R,A,G,ALPHA)
```

Argument Definitions

INPUTS

U(N*(N+1)/2)	Upper triangular vector stored input matrix. D elements are stored on the diagonal. The U vector corresponds to an a priori covariance. If state estimates are involved the last column of U contains X. In this case Dim U = (N+1)*(N+2)/2 and on output (U((N+1)*(N+2)/2) = z-A**T*X(a priori est).
N	Dimension of the state vector
R	Measurement variance
A(N)	Vector of Measurement coefficients; if data then A(N+1) = z
ALPHA	If ALPHA.LT.zero no estimates are computed (and X and z need not be included)

OUTPUTS

U	Updated vector stored U-D factors. When ALPHA (input) is nonnegative the (N+1)st column contains the updated estimate and the predicted residual.
ALPHA	Innovations variance of the measurement residual.
A	Contains U**T*A(input) and when ALPHA (input) is nonnegative A(N+1) = z-A**T*X(a priori est)/ALPHA.

G(N)	Vector of unweighted Kalman gains, K = G/ALPHA.

Remarks and Restrictions

One can use this algorithm with R negative to delete a previously processed data point. One should, however, note that data deletion sometimes introduces numerical errors.

The algorithm holds for R = 0 (a perfect measurement) but the code may fail (zero divides occur) if any of the ALPHA terms appearing in the code vanish. Changes in the code which remove the zero divide problems are commented in the code.

Functional Description

The algorithm updates the columns of U, from left to right, using Bierman's algorithm, cf Proc. 1975 Conf. Dec. Control, Houston, Texas, pp 337-346.

19. UD2COV (U-D factor to covariance)

Purpose

To obtain a covariance from its U-D factorization. Both matrices are vector stored and the output covariance can overwrite the input U-D array. U-D and P are related via $P = UDU^T$.

```
CALL UD2COV(UIN,N,POUT)
```

Argument Definitions

UIN(N*(N+1)/2)	Input vector stored U-D factors, with D entries stored on the diagonal.
POUT(N*(N+1)/2)	Output vector stored covariance matrix (POUT = UIN is permitted).
N	Dimension of the matrices involved.

20. UD2SIG (U-D factors to sigmas)

Purpose

To compute variances from the U-D factors of a matrix.

```
CALL UD2SIG(U,N,SIG,TEXT,NCT)
```

Argument Definitions

U(N*(N+1)/2)	Input vector stored array containing the U-D factors. The D (diagonal) elements are stored on the diagonal of U.
N	Dimension of the U matrix
SIG(N)	Output vector of standard deviations
TEXT ( )	Output label of field data characters, which precedes the printed vector of standard deviations.
NCT	Number of characters of text, 0.LE.NCT.LE.126. If NCT = 0, no sigmas are printed, i.e. nothing is printed.

Functional Description

If U and D are written as doubly subscripted matrices then

$$SIG(J) = \left(D(J,J) + \sum_{K=J+1}^{N} D(K,K)[U(J,K)]^2 \right)^{\frac{1}{2}}$$

If NCT.GT.0 a title is printed, followed by the sigmas.

21. UTINV (Upper triangular matrix inverse)

Purpose

To invert an upper triangular vector stored matrix and store the result in vector form. The algorithm is so arranged that the result can overwrite the input.

```
CALL UTINV(RIN,N,ROUT)
```

Argument Definitions

RIN(N*(N+1)/2)	Input vector stored upper triangular matrix
N	Matrix dimension
ROUT(N*(N+1)/2)	Output vector stored upper triangular matrix inverse (ROUT = RIN is permitted

Remarks and Restrictions

Ill conditioning is not tested, but for nonsingular systems the result is as accurate as is the full rank singular value decomposition inverse. Singularity occurs if a diagonal is zero. The subroutine terminates when it reaches a zero diagonal. The columns to the left of the zero diagonal are, however, inverted and the result stored in ROUT.

This routine can also be used to produce the solution to RX = Z. Place Z in column N+1 (viz. RIN(N*(N+1)/2+1) = Z(1), etc.), define RIN((N+1)(N+2)/2) = -1 and call the subroutine using N+1 instead of N. On return the first N entries of column N+1 contain the solution (e.g. ROUT(N*(N+1)/2+1) = X(1), etc.).

Because matrix inversion is numerically sensitive we recommend using this subroutine only in double precision.

Functional Description

The matrix inversion is accomplished using the standard back substitution method for inverting triangular matrices, cf. the book references by Lawson and Hanson, [1] or Bierman [3].

22. UTIROW (Upper triangular inverse, inverting only the upper rows)

Purpose

To compute the inverse of a vector stored upper triangular matrix, when the lower right corner triangular inverse is given.

```
CALL UTIROW(RIN,N,ROUT,NRY)
```

Argument Definitions

RIN(N*(N+1)/2)	Input vector stored upper triangular matrix. Only the first N - NRY rows are altered by the algorithm.
N	Matrix dimension.
ROUT(N*(N+1)/2)	Output vector stored upper triangular matrix inverse. On input the lower NRY dimensional right corner contains the given (known) inverse. This lower right corner matrix is left unchanged. (ROUT = RIN is permitted.)
NRY	Number of rows, starting at the bottom, that are assumed already inverted.

Remarks and Restrictions

The purpose of this subroutine is to complete the computation of an upper triangular matrix inverse, given that the lower right corner has already been inverted. Part of the input, the rows to be inverted, are inserted via the matrix RIN. The portion of the matrix that has already been inverted is entered via the matrix ROUT. It may seem odd that part of the input matrix is put into RIN and part into ROUT. The reasoning behind this decision is that RIN represents the input matrix to be inverted (it just happens that we do not make use of the lower right triangular entries); ROUT represents the inversion result, and therefore that portion of the inversion that is given should be entered in this array.

Ill conditioning is not tested, but for nonsingular systems the result is accurate. Singularity halts the algorithm if any of the first N-NRY diagonal elements is zero. If the first zero encountered moving up the diagonal (starting at N-NRY) is at diagonal j then the rows below this element will be correctly represented in ROUT.

To generate estimates do the following: put N+1 into the matrix dimension argument; in the first N-NRY rows of the last column of RIN put the right hand side elements of the equation $R_x x + R_{xy} y = z_x$ (i.e., R_x, R_{xy}, and z_x make up the first N-NRY rows of RIN); in the next NRY entries of ROUT, beginning in the (N-NRY+1)st element, put y_{est} (i.e., R_y^{-1} and y_{est} make up rows N-NRY+1,...,N of ROUT); and ROUT((N+1)(N+2)/2) = -1. On output, the last column of ROUT will contain x_{est}, y_{est} and -1.

When NRY = 0 this algorithm is equivalent to subroutine UTINV.

Functional Description

The matrix inversion is accomplished using the standard back substitution method. The computations are arranged row-wise, starting at the bottom (from row N-NRY, since it is assumed that the last NRY rows have already been inverted).

23. WGS (Weighted Gram-Schmidt matrix triangularization)

Purpose

To compute a vector stored U-D array from an input rectangular matrix W, and a diagonal matrix D_W so that $W D_W W^T = UDU^T$.

```
CALL WGS(W,IMAXW,IW,JW,DW,U,V)
```

Argument Definitions

W(IW,JW)	Input rectangular matrix, destroyed by the computations
IMAXW	Row dimension of input W matrix, IMAXW.GE.IW
DW(JW)	Diagonal input matrix; the entries are assumed to be nonnegative. This vector is unaltered by the computations
U(IW*(IW+1)/2)	Vector stored output U-D array
V(JW)	Work vector in the computation

Remarks and Restrictions

The algorithm is not numerically stable when negative DW weights are used; negative weights are, however, allowed. If JW is less than IW (more rows than columns), the output U-D array is singular; with IW-JW zero diagonal entries in the output U array.

Functional Description

A D_W-orthogonal set of row vectors, ϕ_1, ϕ_2,..., ϕ_{IW}, are constructed from the input rows of the W matrix, i.e., $W = U\ \phi$, , $\phi D_W \phi^T = D$. The construction is accomplished using the modified Gram-Schmidt orthogonal construction (see refs. [1] or [3]). This algorithm is reputed to have excellent numerical properties. Note that the ϕ vectors are not of interest in this routine, and they are overwritten; The V vector used in the program houses vector IW-j+1 of ϕ at step j of algorithm. The fact that the computed ϕ vectors may not be D orthogonal is of no import in regard to the U and D computed results.

V. FORTRAN Subroutine Listings

```
      SUBROUTINE AGTRN (UIN,UOUT,N,C,V)                                 AGTRN010
C                                                                       AGTRN020
C         AGEE-TURNER U-D FACTOR RANK 1 UPDATE                          AGTRN030
C                                                                       AGTRN040
C              (UOUT)*DOUT*(UOUT)**T=(UIN)*DIN*(UIN)**T+C*V*V**T        AGTRN050
C                                                                       AGTRN060
C     UIN(N*(N+1)/2)  INPUT VECTOR STORED POSITIVE SEMI-DEFINITE U-D    AGTRN070
C                     ARRAY, WITH D ELEMENTS STORED ON THE DIAGONAL     AGTRN080
C     UOUT(N*(N+1)/2) OUTPUT VECTOR STORED POSITIVE (POSSIBLY) SEMI-    AGTRN090
C                     DEFINITE U-D RESULT. UOUT=UIN IS PERMITTED        AGTRN100
C     N               DIMENSION OF THE STATE                            AGTRN110
C     C               SCALAR. SHOULD BE NON-NEGATIVE                    AGTRN120
C                     C IS DESTROYED DURING THE PROCESS                 AGTRN130
C     V(N)            INPUT VECTOR FOR RANK ONE MODIFICATION. V IS      AGTRN140
C                     DESTROYED DURING THE PROCESS                      AGTRN150
C                                                                       AGTRN160
C     COGNIZANT PERSONS: G.J.BIERMAN/M.W.NEAD   (JPL,FEB.1977)          AGTRN170
C                                                                       AGTRN180
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               AGTRN190
      DIMENSION     UIN(1),   UOUT(1),  V(1)                            AGTRN200
C                                                                       AGTRN210
      Z=0.0                                                             AGTRN220
      IF (C.EQ.Z) RETURN                                                AGTRN230
C                                                                       AGTRN240
      JJ=N*(N+1)/2                                                      AGTRN250
      DO 50 J=N,2,-1                                                    AGTRN260
        S=V(J)                                                          AGTRN270
        D=UIN(JJ)+C*S*S                                                 AGTRN280
        IF (D) 5,10,30                                                  AGTRN290
    5   WRITE (6,100)                                                   AGTRN300
        RETURN                                                          AGTRN310
   10   JJ=JJ-J                                                         AGTRN320
        WRITE (6,110)                                                   AGTRN330
        DO 20 K=1,J                                                     AGTRN340
   20   UOUT(JJ+K)=Z                                                    AGTRN350
        GO TO 50                                                        AGTRN360
   30   B=C/D                                                           AGTRN370
        BETA=S*B                                                        AGTRN380
        C=B*UIN(JJ)                                                     AGTRN390
        UOUT(JJ)=D                                                      AGTRN400
        JJ=JJ-J                                                         AGTRN410
        JM1=J-1                                                         AGTRN420
        DO 40 I=1,JM1                                                   AGTRN430
          V(I)=V(I)-S*UIN(JJ+I)                                         AGTRN440
   40     UOUT(JJ+I)=UIN(JJ+I)+BETA*V(I)                                AGTRN450
   50   CONTINUE                                                        AGTRN460
C                                                                       AGTRN470
      UOUT(1)=UIN(1)+C*V(1)**2                                          AGTRN480
      RETURN                                                            AGTRN490
C                                                                       AGTRN500
  100 FORMAT (1H0,10X,'* * * ERROR RETURN DUE TO A COMPUTED NEGATIVE COMAGTRN510
     1PUTED DIAGONAL IN AGTRN * * *')                                   AGTRN520
  110 FORMAT (1H0,10X,'* * * NOTE: U-D RESULT IS SINGULAR * * *')       AGTRN530
      END                                                               AGTRN540
```

```
      SUBROUTINE A2A1 (A,IA,IR,LA,NAMA,A1,IA1,LA1,NAMA1)
C                                                                       A2A10010
C        SUBROUTINE TO REARRANGE THE COLUMNS OF A(IR,LA), IN NAMA ORDER A2A10020
C        AND PUT THE RESULT IN A1(IR,LA1) IN NAMA1 ORDER.  ZERO COLUMNS A2A10030
C        ARE INSERTED IN A1 CORRESPONDING TO THE NEWLY DEFINED NAMES.   A2A10040
C                                                                       A2A10050
C        A(IR,LA)     INPUT RECTANGULAR MATRIX                          A2A10060
C        IA           ROW DIMENSION OF A, IR.LE.IA                      A2A10070
C        IR           NO. OF ROWS OF A THAT ARE TO BE REARRANGED        A2A10080
C        LA           NO. OF PARAMETER NAMES ASSOCIATED WITH A          A2A10090
C        NAMA(LA)     PARAMETER NAMES ASSOCIATED WITH A                 A2A10100
C        A1(IR,LA1)   OUTPUT RECTANGULAR MATRIX                         A2A10110
C                     A AND A1 CANNOT SHARE COMMON STORAGE              A2A10120
C        IA1          ROW DIMENSION OF A1, IR.LE.IA1                    A2A10130
C        LA1          NO. OF PARAMETER NAMES ASSOCIATED WITH A1         A2A10140
C        NAMA1(LA1)   INPUT LIST OF PARAMETER NAMES TO BE ASSOCIATED    A2A10150
C                     WITH THE OUTPUT MATRIX A1                         A2A10160
C                                                                       A2A10170
C        COGNIZANT PERSONS:  G.J.BIERMAN/M.W.NEAD (JPL, SEPT. 1976)     A2A10180
C                                                                       A2A10190
      DIMENSION A(IA,1), NAMA(1), A1(IA1,1),NAMA1(1)                    A2A10200
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               A2A10210
C                                                                       A2A10220
      ZERO=0.                                                           A2A10230
      DO 100 J=1,LA1                                                    A2A10240
        DO 60 I=1,LA                                                    A2A10250
          IF (NAMA(I).EQ.NAMA1(J)) GO TO 80                             A2A10260
   60     CONTINUE                                                      A2A10270
        DO 70 K=1,IR                                                    A2A10280
   70     A1(K,J)=ZERO       @ ZERO COL. CORRES. TO NEW NAME           A2A10290
        GO TO 100                                                       A2A10300
   80   DO 90 K=1,IR                                                    A2A10310
   90     A1(K,J)=A(K,I)     @ COPY COL. ASSOC. WITH OLD NAME          A2A10320
  100   CONTINUE                                                        A2A10330
C                                                                       A2A10340
      RETURN                                                            A2A10350
      END                                                               A2A10360
```

```
      SUBROUTINE COMBO (R,L1,NAM1,L2,NAM2,A,IA,LA,NAMA)
C
C         TO REARRANGE A VECTOR STORED TRIANGULAR MATRIX AND STORE      COMBO010
C         THE RESULT IN MATRIX A.  THE DIFFERENCE BETWEEN THIS SUB-     COMBO020
C         ROUTINE AND R2A IS THAT THERE THE NAMELIST FOR A IS INPUT.    COMBO030
C         HERE IT IS DETERMINED BY COMBINING THE LIST FOR R WITH        COMBO040
C         A LIST OF DESIRED NAMES.                                      COMBO050
C                                                                       COMBO060
C        R(L1*(L1+1)/2)  INPUT VECTOR STORED UPPER TRIANGULAR MATRIX    COMBO070
C        L1              NO. OF PARAMETERS IN R (AND IN NAM1)           COMBO080
C        NAM1(L1)        NAMES ASSOCIATED WITH R                        COMBO090
C        L2              NO. OF PARAMETERS IN NAM2                      COMBO100
C        NAM2(L2)        PARAMETER NAMES THAT ARE TO BE COMBINED WITH R COMBO110
C                        (NAM1 LIST). THESE NAMES MAY OR MAY NOT BE IN  COMBO120
C                        NAM1.                                          COMBO130
C        A(L1,LA)        OUTPUT ARRAY CONTAINING THE REARRANGED         COMBO140
C                        R MATRIX, L1.LE.IA.                            COMBO150
C        IA              ROW DIMENSION OF A                             COMBO160
C        LA              NO. OF PARAMETER NAMES IN NAMA, AND THE        COMBO170
C                        COLUMN DIMENSION OF A.  LA=L1+L2-NO. NAMES     COMBO180
C                        COMMON TO NAM1 AND NAM2.  LA IS COMPUTED AND   COMBO190
C                        OUTPUT.                                        COMBO200
C        NAMA(LA)        PARAMETER NAMES ASSOCIATED WITH THE OUTPUT A   COMBO210
C                        MATRIX.  CONSISTS OF NAMES IN NAM1 NOT IN      COMBO220
C                        NAM2 FOLLOWED BY NAM2.                         COMBO230
C                                                                       COMBO240
C        COGNIZANT PERSONS:  G.J.BIERMAN/M.W.NEAD (JPL, SEPT. 1976)     COMBO250
C                                                                       COMBO260
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               COMBO270
      DIMENSION  R(1), A(IA,1), NAM1(1), NAM2(1), NAMA(1)               COMBO280
C                                                                       COMBO290
      ZERO=0.0                                                          COMBO300
      K=1                                                               COMBO310
      DO 100 I=1,L1                                                     COMBO320
        DO 50 J=1,L2                                                    COMBO330
          IF (NAM1(I).EQ.NAM2(J)) GO TO 100                             COMBO340
   50     CONTINUE                                                      COMBO350
        NAMA(K)=NAM1(I)                                                 COMBO360
      JJ=I*(I-1)/2                                                      COMBO370
        DO 60 L=1,I                                                     COMBO380
   60     A(L,K)=R(JJ+L)                                                COMBO390
      IF (I.EQ.L1) GO TO 80                                             COMBO400
      IP1 = I+1                                                         COMBO410
      DO 70 L=IP1,L1                                                    COMBO420
   70 A(L,K) = ZERO                                                     COMBO430
   80   K=K+1                                                          COMBO440
  100   CONTINUE                                                       COMBO450
C               NAMES UNIQUE TO NAM1 ARE NOW IN NAMA                    COMBO460
      DO 200 J=1,L2                                                     COMBO470
        DO 150 I=1,L1                                                   COMBO480
          IF (NAM2(J).EQ.NAM1(I)) GO TO 170                             COMBO490
  150     CONTINUE                                                      COMBO500
        NAMA(K)=NAM2(J)                                                 COMBO510
        DO 160 L=1,L1                                                   COMBO520
  160     A(L,K)=ZERO                                                   COMBO530
```

```
C               NAMES UNIQUE TO NAM2 ARE NOW IN NAMA                    COMB0540
        GO TO 190                                                       COMB0550
  170   NAMA(K)=NAM2(J)                                                 COMB0560
C          LOCATE DIAGONAL OF PRECEDING COLUMN                          COMB0570
      JJ=I*(I-1)/2                                                      COMB0580
        DO 180 L=1,I                                                    COMB0590
  180     A(L,K)=R(JJ+L)                                                COMB0600
      IF (I.EQ.L1) GO TO 190                                            COMB0610
      IP1=I+1                                                           COMB0620
      DO 185 L=IP1,L1                                                   COMB0630
  185 A(L,K)=ZERO                                                       COMB0640
  190   K=K+1                                                           COMB0650
  200   CONTINUE                                                        COMB0660
      LA=K-1                                                            COMB0670
C               NAMES MUTUAL TO NAM1 AND NAM2 ARE NOW IN NAMA           COMB0680
      RETURN                                                            COMB0690
      END                                                               COMB0700
```

```
      SUBROUTINE COV2RI(U,N)
C                                                                       COV2R010
C        TO CONSTRUCT THE UPPER TRIANGULAR CHOLESKY FACTOR OF A         COV2R020
C        POSITIVE SEMI-DEFINITE MATRIX. BOTH THE INPUT COVARIANCE       COV2R030
C        AND THE OUTPUT CHOLESKY FACTOR (SQUARE ROOT) ARE VECTOR        COV2R040
C        STORED. THE OUTPUT OVERWRITES THE INPUT.                       COV2R050
C        COVARIANCE(INPUT)=U*U**T  (U IS OUTPUT).                       COV2R060
C                                                                       COV2R070
C        IF THE INPUT COVARIANCE IS SINGULAR THE OUTPUT FACTOR HAS      COV2R080
C        ZERO COLUMNS.                                                  COV2R090
C                                                                       COV2R100
C        U(N*(N+1)/2)  CONTAINS THE INPUT VECTOR STORED COVARIANCE      COV2R110
C                      MATRIX (ASSUMED POSITIVE DEFINITE) AND ON OUTPUT COV2R120
C                      IT CONTAINS THE UPPER TRIANGULAR SQUARE ROOT     COV2R130
C                      FACTOR.                                          COV2R140
C        N             DIMENSION OF THE MATRICES INVOLVED               COV2R150
C                                                                       COV2R160
C        COGNIZANT PERSONS:  G.J.BIERMAN/M.W.MEAD (JPL, FEB. 1977)      COV2R170
C                                                                       COV2R180
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               COV2R190
      DIMENSION U(1)                                                    COV2R200
C                                                                       COV2R210
      ZERO=0.0                                                          COV2R220
      ONE=1.                                                            COV2R230
      JJ=N*(N+1)/2                                                      COV2R240
      JJN=JJ                                                            COV2R250
C                                                                       COV2R260
      DO 5 J=N,2,-1                                                     COV2R270
          IF (U(JJ).LT.ZERO) U(JJ)=ZERO                                 COV2R280
          U(JJ)= SQRT(U(JJ))                                            COV2R290
          IF (U(JJ).GT.ZERO) ALPHA=ONE/U(JJ)                            COV2R300
C                                                                       COV2R310
          KK=0                                                          COV2R320
          JJN=JJ-J                          @ NEXT DIAGONAL             COV2R330
          JM1=J-1                                                       COV2R340
            DO 4 K=1,JM1                                                COV2R350
              U(JJN+K)=ALPHA*U(JJN+K)       @ JJN+K=(K,J)               COV2R360
              S=U(JJN+K)                                                COV2R370
              DO 3 I=1,K                                                COV2R380
    3           U(KK+I)=U(KK+I)-S*U(JJN+I)  @ KK+I=(I,K)                COV2R390
    4         KK=KK+K                                                   COV2R400
    5 JJ=JJN                                                            COV2R410
      IF (U(1).LT.ZERO) U(1)=ZERO                                       COV2R420
      U(1)= SQRT(U(1))                                                  COV2R430
C                                                                       COV2R440
      RETURN                                                            COV2R450
      END                                                               COV2R460
```

```
      SUBROUTINE COV2UD (U,N)
C                                                                       COV2U010
C        TO OBTAIN THE U-D FACTORS OF A POSITIVE SEMI-DEFINITE MATRIX.  COV2U020
C        THE INPUT MATRIX VECTOR STORED IS OVERWRITTEN BY THE OUTPUT    COV2U030
C        U-D FACTORS WHICH ARE ALSO VECTOR STORED.                      COV2U040
C                                                                       COV2U050
C        U(N*(N+1)/2)  CONTAINS INPUT VECTOR STORED COVARIANCE MATRIX.  COV2U060
C                      ON OUTPUT IT CONTAINS THE VECTOR STORED U-D       COV2U070
C                      COVARIANCE FACTORS.                              COV2U080
C        N             MATRIX DIMENSION                                 COV2U090
C                                                                       COV2U100
C        SINGULAR INPUT COVARIANCES RESULT IN OUTPUT MATRICES WITH ZERO  COV2U110
C        COLUMNS                                                        COV2U120
C                                                                       COV2U130
C                                                                       COV2U140
C        COGNIZANT PERSONS:  G.J.BIERMAN/R.A.JACOBSON (JPL, FEB. 1977)  COV2U150
C                                                                       COV2U160
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               COV2U170
C                                                                       COV2U180
      DIMENSION U(1)                                                    COV2U190
C                                                                       COV2U200
      Z=0.0                                                             COV2U210
      ONE=1.0                                                           COV2U220
C                                                                       COV2U230
      JJ=N*(N+1)/2                                                      COV2U240
      DO 50 J=N,2,-1                                                    COV2U250
        ALPHA=Z                                                         COV2U260
        IF (U(JJ).LT.Z) U(JJ)=Z                                         COV2U270
        IF (U(JJ).GT.Z) ALPHA=ONE/U(JJ)                                 COV2U280
        JJ=JJ-J                                                         COV2U290
        KK=0                                                            COV2U300
        KJ=JJ                                                           COV2U310
        JM1=J-1                                                         COV2U320
        DO 40 K=1,JM1                                                   COV2U330
          KJ=KJ+1                                                       COV2U340
          BETA=U(KJ)                                                    COV2U350
          U(KJ)=ALPHA*U(KJ)                                             COV2U360
          IJ=JJ                                                         COV2U370
          IK=KK                                                         COV2U380
          DO 30 I=1,K                                                   COV2U390
            IK=IK+1                                                     COV2U400
            IJ=IJ+1                                                     COV2U410
   30       U(IK)=U(IK)-BETA*U(IJ)                                      COV2U420
   40     KK=KK+K                                                      COV2U430
   50   CONTINUE                                                       COV2U440
      IF (U(1).LT.7) U(1)=7                                             COV2U450
      RETURN                                                            COV2U460
      END                                                               COV2U470
```

```
      SUBROUTINE C2C (C,IC,L1,NAM1,L2,NAM2)
C                                                                       C2C00010
C        SUBROUTINE TO REARRANGE THE ROWS AND COLUMNS OF MATRIX         C2C00020
C        C(L1,L1) IN NAM1 ORDER AND PUT THE RESULT IN                   C2C00030
C        C(L2,L2) IN NAM2 ORDER. ZERO COLUMNS AND ROWS ARE              C2C00040
C        ASSOCIATED WITH OUTPUT DEFINED NAMES THAT ARE NOT CONTAINED    C2C00050
C        IN NAM1.                                                       C2C00060
C                                                                       C2C00070
C        C(L1,L1)     INPUT MATRIX                                      C2C00080
C        IC           ROW DIMENSION OF C, IC.GE.L=MAX(L1,L2)            C2C00090
C        L1           NO. OF PARAMETER NAMES ASSOCIATED WITH THE INPUT C C2C00100
C        NAM1(L)      PARAMETER NAMES ASSOCIATED WITH C ON INPUT. (ONLY  C2C00110
C                     THE FIRST L1 ENTRIES APPLY TO THE INPUT C)        C2C00120
C        L2           NO. OF PARAMETER NAMES ASSOCIATED WITH THE OUTPUT CC2C00130
C        NAM2(L2)     PARAMETER NAMES ASSOCIATED WITH THE OUTPUT C       C2C00140
C                                                                       C2C00150
C        COGNIZANT PERSONS:  G.J.BIERMAN/M.W.NEAD (JPL, SEPT. 1976)     C2C00160
C                                                                       C2C00170
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               C2C00180
      DIMENSION  C(IC,1), NAM1(1), NAM2(1)                              C2C00190
C                                                                       C2C00200
      ZERO=0.                                                           C2C00210
      L=MAX(L1,L2)                                                      C2C00220
      IF (L.LE.L1) GO TO 5                                              C2C00230
      NM=L1+1                                                           C2C00240
      DO 1 K=NM,L                                                       C2C00250
    1   NAM1(K)= ZERO          @ ZERO REMAINING NAM1 LOCNS              C2C00260
    5 DO 90 J=1,L2                                                      C2C00270
        DO 10 I=1,L                                                     C2C00280
          IF (NAM1(I).EQ.NAM2(J)) GO TO 30                              C2C00290
   10     CONTINUE                                                      C2C00300
        GO TO 90                                                        C2C00310
   30    IF (I.EQ.J) GO TO 90                                           C2C00320
        DO 40 K=1,L                                                     C2C00330
          H=C(K,J)             @ INTERCHANGE COLUMNS I AND J            C2C00340
          C(K,J)=C(K,I)                                                 C2C00350
   40     C(K,I)=H                                                      C2C00360
        DO 80 K=1,L                                                     C2C00370
          H=C(J,K)             @ INTERCHANGE ROWS I AND J               C2C00380
          C(J,K)=C(I,K)                                                 C2C00390
   80     C(I,K)=H                                                      C2C00400
          NM=NAM1(I)           @ INTERCHANGE LABELS I AND J             C2C00410
          NAM1(I)=NAM1(J)                                               C2C00420
          NAM1(J)=NM                                                    C2C00430
   90   CONTINUE                                                        C2C00440
C                                                                       C2C00450
C         FIND NAM2 NAMES NOT IN NAM1 AND SET CORRESPONDING ROWS AND    C2C00460
C         COLUMNS TO ZERO                                               C2C00470
C                                                                       C2C00480
      DO 120 J=1,L2                                                     C2C00490
        DO 100 I=1,L                                                    C2C00500
          IF (NAM1(I).EQ.NAM2(J)) GO TO 120                             C2C00510
  100     CONTINUE                                                      C2C00520
        DO 110 K=1,L2                                                   C2C00530
          C(J,K)=ZERO                                                   C2C00540
  110     C(K,J)=ZERO                                                   C2C00550
  120   CONTINUE                                                        C2C00560
C                                                                       C2C00570
      RETURN                                                            C2C00580
      END                                                               C2C00590
```

```
C
      SUBROUTINE INF2R (P,N)                                                INF2R010
C                                                                           INF2R020
C         TO CHOLESKY FACTOR AN INFORMATION MATRIX                          INF2R030
C                                                                           INF2R040
C     COMPUTES A LOWER TRIANGULAR VECTOR STORED CHOLESKY FACTORIZATION      INF2R050
C     OF A POSITIVE SEMI-DFFINITE MATRIX. P=R(**T)R, R UPPER TRIANGULAR.    INF2R060
C     BOTH MATRICES ARE VECTOR STORED AND THE RESULTS OVERWRITES            INF2R070
C     THE INPUT                                                             INF2R080
C                                                                           INF2R090
C     P(N*(N+1)/2)  ON INPUT THIS IS A POSITIVE SEMI-DEFINITE MATRIX,       INF2R100
C                   AND ON OUTPUT IT IS A TRIANGULAR FACTOR. IF THE         INF2R110
C                   INPUT MATRIX IS SINGULAR THE OUTPUT MATRIX WILL         INF2R120
C                   HAVE ZERO DIAGONAL ENTRIES                              INF2R130
C     N             DIMENSION OF MATRICES INVOLVED                          INF2R140
C                                                                           INF2R150
C     COGNIZANT PERSON: G.J.BIERMAN/M.W.MEAD      (JPL,FEB.1977)            INF2R160
C                                                                           INF2R170
      IMPLICIT DOUBLE PRECISION  (A-H,O-Z)                                  INF2R180
C                                                                           INF2R190
      DIMENSION P(1)                                                        INF2R200
C                                                                           INF2R210
      Z=0.0                                                                 INF2R220
      ONE=1.0                                                               INF2R230
      JJ=0                                                                  INF2R240
      NN=N*(N+1)/2                                                          INF2R250
      NM1=N-1                                                               INF2R260
      DO 10 J=1,NM1                                                         INF2R270
        JJ=JJ+J                              @ JJ=(J,J)                   INF2R280
        IF (P(JJ).LT.Z)  P(JJ)=Z                                            INF2R290
        P(JJ)=SQRT(P(JJ))                                                   INF2R300
        ALPHA=Z                                                             INF2R310
        IF (P(JJ).GT.Z) ALPHA=ONE/P(JJ)                                     INF2R320
        JK=NN+J                              @ JK=(J,K)                   INF2R330
        JP1=J+1                                                             INF2R340
        JIS=JK                               @ JIS=(J,I) START            INF2R350
        DO 10 K=N,JP1,-1                                                    INF2R360
          JK=JK-K                                                           INF2R370
          P(JK)=ALPHA*P(JK)                                                 INF2R380
          BETA=P(JK)                                                        INF2R390
          KI=NN+K                                                           INF2R400
          JI=JIS                                                            INF2R410
          DO 10 I=N,K,-1                                                    INF2R420
            KI=KI-I                                                         INF2R430
            JI=JI-I                                                         INF2R440
   10       P(KI)=P(KI)-P(JI)*BETA                                          INF2R450
C                                                                           INF2R460
      IF (P(NN).LT.Z) P(NN)=Z                                               INF2R470
      P(NN)=SQRT(P(NN))                                                     INF2R480
      RETURN                                                                INF2R490
      END                                                                   INF2R500
```

```
      SUBROUTINE PERMUT (A,IA,IR,L1,NAM1,L2,NAM2)
C                                                                       PERMU010
C        SUBROUTINE TO REARRANGE PARAMETERS OF A(IR,L1), NAM1 ORDER     PERMU020
C        TO A(IR,L2), NAM2 ORDER.  ZERO COLUMNS ARE INSERTED            PERMU030
C        CORRESPONDING TO THE NEWLY DEFINED NAMES.                      PERMU040
C                                                                       PERMU050
C        A(IR,L)    INPUT RECTANGULAR MATRIX, L=MAX(L1,L2)              PERMU060
C        IA         ROW DIMENSION OF A, IA.GE.IR                        PERMU070
C        IR         NUMBER OF ROWS OF A THAT ARE TO BE REARRANGED       PERMU080
C        L1         NUMBER OF PARAMETER NAMES ASSOCIATED WITH THE INPUT PERMU090
C                   A MATRIX                                            PERMU100
C        NAM1(L)    PARAMETER NAMES ASSOCIATED WITH A ON INPUT          PERMU110
C                   (ONLY THE FIRST L1 ENTRIES APPLY TO THE INPUT A)    PERMU120
C        L2         NUMBER OF PARAMETER NAMES ASSOCIATED WITH THE OUTPUT PERMU130
C                   A MATRIX                                            PERMU140
C        NAM2       PARAMETER NAMES ASSOCIATED WITH THE OUTPUT A        PERMU150
C                                                                       PERMU160
C        COGNIZANT PERSONS:  G.J.BIERMAN/M.W.NEAD (JPL, SEPT. 1976)     PERMU170
C                                                                       PERMU180
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               PERMU190
      DIMENSION A(IA,1), NAM1(1), NAM2(1)                               PERMU200
C                                                                       PERMU210
      ZERO=0.                                                           PERMU220
      L=MAX(L1,L2)                                                      PERMU230
      IF (L.LE.L1) GO TO 50                                             PERMU240
      NM=L1+1                                                           PERMU250
      DO 40 K=NM,L                                                      PERMU260
   40   NAM1(K)=0          @ ZERO REMAINING NAM1 LOCS                   PERMU270
   50 DO 100 J=1,L2                                                     PERMU280
        DO 60 I=1,L                                                     PERMU290
          IF (NAM1(I).EQ.NAM2(J)) GO TO 65                              PERMU300
   60     CONTINUE                                                      PERMU310
      GO TO 100                                                         PERMU320
   65 CONTINUE                                                          PERMU330
        IF (I.EQ.J) GO TO 100                                           PERMU340
        DO 70 K=1,IR       @ INTERCHANGE COLS I AND J                   PERMU350
          W=A(K,J)                                                      PERMU360
          A(K,J)=A(K,I)                                                 PERMU370
   70     A(K,I)=W                                                      PERMU380
          NM=NAM1(I)       @ INTERCHANGE I AND J COL. LABELS            PERMU390
          NAM1(I)=NAM1(J)                                               PERMU400
          NAM1(J)=NM                                                    PERMU410
  100   CONTINUE                                                        PERMU420
C                       REPEAT TO FILL NEW COLS                         PERMU430
        DO 200 J=1,L2                                                   PERMU440
          DO 160 I=1,L                                                  PERMU450
            IF (NAM1(I).EQ.NAM2(J)) GO TO 200                           PERMU460
  160       CONTINUE                                                    PERMU470
          DO 170 K=1,IR                                                 PERMU480
  170       A(K,J)=ZERO                                                 PERMU490
  200     CONTINUE                                                      PERMU500
C                                                                       PERMU510
      RETURN                                                            PERMU520
      END                                                               PERMU530
```

```
C
      SUBROUTINE RINCON (RIN,N,ROUT,CNB)                                RINC0010
C                                                                       RINC0020
C     TO COMPUTE THE INVERSE OF THE UPPER TRIANGULAR VECTOR STORED      RINC0030
C     INPUT MATRIX RIN AND STORE THE RESULT IN ROUT. (RIN=ROUT IS       RINC0040
C     PERMITTED) AND TO COMPUTE A CONDITION NUMBER ESTIMATE.            RINC0050
C     CNB=FROB.NORM(R)*FROB.NORM(R**-1).                                RINC0060
C     THE FROBENIUS NORM IS THE SQUARE ROOT OF THE SUM OF SQUARES       RINC0070
C     OF THE ELEMENTS. THIS CONDITION NUMBER BOUND IS USED AS           RINC0080
C     AN UPPER BOUND AND IT ACTS AS A LOWER BOUND ON THE ACTUAL         RINC0090
C     CONDITION NUMBER OF THE PROBLEM. (SEE THE BOOK 'SOLVING LEAST     RINC0100
C     SQUARES', BY LAWSON AND HANSON)                                   RINC0110
C                                                                       RINC0120
C     RIN(N*(N+1)/2)  INPUT VECTOR STORED UPPER TRIANGULAR MATRIX       RINC0130
C     N               DIMENSION OF R MATRICES                           RINC0140
C     ROUT(N*(N+1)/2) OUTPUT VECTOR STORED UPPER TRIANGULAR MATRIX      RINC0150
C                     INVERSE (RIN=ROUT IS PERMITTED)                   RINC0160
C     CNB             CONDITION NUMBER BOUND                            RINC0170
C                                                                       RINC0180
C     COGNIZANT PERSONS: G.J.BIERMAN/M.W.MEAD   (JPL,FEB.1977)          RINC0190
C                                                                       RINC0200
C     SUBROUTINES REQUIRED:  UTINV                                      RINC0210
C                                                                       RINC0220
      IMPLICIT DOUBLE PRECISION  (A-H,O-Z)                              RINC0230
      DIMENSION RIN(1), ROUT(1)                                         RINC0240
C                                                                       RINC0250
      Z=0.0                                                             RINC0260
      NTOT=N*(N+1)/2                                                    RINC0270
C                                                                       RINC0280
      RNM=Z                                                             RINC0290
      DO 10 J=1,NTOT                                                    RINC0300
   10   RNM=RNM+RIN(J)**2                                              RINC0310
C                                                                       RINC0320
      CALL UTINV (RIN,N,ROUT)                                           RINC0330
      RNMOUT=Z                                                          RINC0340
      DO 20 J=1,NTOT                                                    RINC0350
   20   RNMOUT=RNMOUT+ROUT(J)**2                                       RINC0360
C                                                                       RINC0370
      CNB=SQRT(RNM*RNMOUT)                                              RINC0380
C                                                                       RINC0390
      WRITE (6,30) CNB                                                  RINC0400
      RETURN                                                            RINC0410
C                                                                       RINC0420
   30 FORMAT(1H0,5X,'CONDITION NUMBER BOUND=',D18.10,2X,'CNB/N.LE.CONDITRINC0430
     1ION NUMBER.LE.CNB',/)                                             RINC0440
      END                                                               RINC0450
```

```
      SUBROUTINE RI2COV (RINV,N,SIG,COVOUT,KOV)
C                                                                       RI2C0010
C        TO COMPUTE THE COVARIANCE MATRIX AND/OR THE STANDARD DEVIATIONSRI2C0020
C        OF A VECTOR STORED UPPER TRIANGULAR SQUARE ROOT COVARIANCE     RI2C0030
C        MATRIX.  THE OUTPUT COVARIANCE MATRIX IS ALSO VECTOR STORED.   RI2C0040
C                                                                       RI2C0050
C        RINV(N*(N+1)/2)   INPUT VECTOR STORED UPPER TRIANGULAR COVARI- RI2C0060
C                          ANCE SQUARE ROOT. (RINV=R INVERSE IS THE     RI2C0070
C                          INVERSE OF THE SRIF MATRIX)                  RI2C0080
C        N                 DIMENSION OF THE RINV MATRIX                 RI2C0090
C        SIG(N)            OUTPUT VECTOR OF STANDARD DEVIATIONS         RI2C0100
C        COVOUT(N*(N+1)/2) OUTPUT VECTOR STORED COVARIANCE MATRIX       RI2C0110
C                          (COVOUT = RINV IS ALLOWED)                   RI2C0120
C        KOV  .GT.0        COMPUTE COVARIANCE AND SIGMAS USING KOV ROWS RI2C0130
C                          OF RINV.                                     RI2C0140
C             .LT.0        COMPUTE ONLY THE SIGMAS USING KOV ROWS OF    RI2C0150
C                          RINV.                                        RI2C0160
C             .EQ.0        NO COVARIANCE, BUT ALL SIGMAS (E.G. USE      RI2C0170
C                          N ROWS OF RINV).                             RI2C0180
C                                                                       RI2C0190
C        COGNIZANT PERSONS:  G.J.BIERMAN/M.W.NEAD (JPL, SEPT. 1976)     RI2C0200
C                                                                       RI2C0210
      IMPLICIT DOUBLE PRECISION  (A-H,O-Z)                              RI2C0220
      DIMENSION RINV(1), SIG(1), COVOUT(1)                              RI2C0230
C                                                                       RI2C0240
      ZERO=0.0                                                          RI2C0250
      LIM=N                                                             RI2C0260
      IF (KOV.NE.0) LIM=IABS(KOV)                                       RI2C0270
C          *** COMPUTE SIGMAS                                           RI2C0280
      IKS=0                                                             RI2C0290
      DO 2 J=1,LIM                                                      RI2C0300
        IKS=IKS+J                                                       RI2C0310
        SUM=ZERO                                                        RI2C0320
        IK=IKS                                                          RI2C0330
        DO 1 K=J,N                                                      RI2C0340
          SUM=SUM+RINV(IK)**2                                           RI2C0350
    1     IK=IK+K                                                       RI2C0360
    2   SIG(J)=SQRT(SUM)                                                RI2C0370
C                                                                       RI2C0380
      IF (KOV.LE.0) RETURN                                              RI2C0390
C          *** COMPUTE COVARIANCE                                       RI2C0400
      JJ=0                                                              RI2C0410
      NM1=LIM-1                                                         RI2C0420
      DO 10 J=1,NM1                                                     RI2C0430
        JJ=JJ+J                                                         RI2C0440
        COVOUT(JJ)=SIG(J)**2                                            RI2C0450
        IJS=JJ+J                                                        RI2C0460
        JP1=J+1                                                         RI2C0470
        DO 10 I=JP1,N                                                   RI2C0480
          IK=IJS                                                        RI2C0490
          IMJ=I-J                                                       RI2C0500
          SUM=ZERO                                                      RI2C0510
          DO 5 K=I,N                                                    RI2C0520
            IJK=IK+IMJ                                                  RI2C0530
            SUM=SUM+RINV(IK)*RINV(IJK)                                  RI2C0540
    5       IK=IK+K                                                     RI2C0550
          COVOUT(IJS)=SUM                                               RI2C0560
   10     IJS=IJS+I                                                     RI2C0570
      IF (KOV.EQ.N) COVOUT(JJ+N)=SIG(N)**2                              RI2C0580
C                                                                       RI2C0590
      RETURN                                                            RI2C0600
      END                                                               RI2C0610
```

```
      SUBROUTINE R2A(R,LR,NAMR,A,IA,LA,NAMA)
C                                                                       R2A00010
C        TO PLACE THE TRIANGULAR VECTOR STORED MATRIX R INTO THE        R2A00020
C        MATRIX A AND TO ARRANGE THE COLUMNS TO MATCH THE DESIRED       R2A00030
C        NAMA PARAMETER LIST.  NAMES IN THE NAMA LIST THAT DO NOT       R2A00040
C        CORRESPOND TO ANY NAME IN NAMR HAVE ZERO ENTRIES IN THE        R2A00050
C        CORRESPONDING A COLUMN.                                        R2A00060
C                                                                       R2A00070
C        R(LR*(LR+1)/2)  INPUT UPPER TRIANGULAR VECTOR STORED ARRAY     R2A00080
C        LR              DIMENSION OF R                                 R2A00090
C        NAMR(L)         PARAMETER NAMES ASSOCIATED WITH R.  ONLY THE   R2A00100
C                        FIRST LR ENTRIES APPLY TO R, L=MAX(LR,LA).     R2A00110
C        A(IR,LA)        MATRIX TO HOUSE THE REARRANGED R MATRIX        R2A00120
C        IA              ROW DIMENSION OF A, IA.GE.LR                   R2A00130
C        LA              NO. OF PARAMETER NAMES ASSOCIATED WITH THE     R2A00140
C                        OUTPUT A MATRIX                                R2A00150
C        NAMA(LA)        PARAMETER NAMES FOR THE OUTPUT A MATRIX        R2A00160
C                                                                       R2A00170
C        COGNIZANT PERSONS:  G.J.BIERMAN/M.W.MEAD (JPL, SEPT. 1976)     R2A00180
C                                                                       R2A00190
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               R2A00200
      DIMENSION R(1),NAMR(1),A(IA,1),NAMA(1)                            R2A00210
C                                                                       R2A00220
       ZERO=0.                                                          R2A00230
      DO 5 J=1,LA                                                       R2A00240
      DO 5 K=1,LR                                                       R2A00250
    5 A(K,J)=ZERO          @ ZERO A(LR,LA)                            R2A00260
      DO 40 J=1,LA                                                      R2A00270
        DO 10 I=1,LR                                                    R2A00280
          IF (NAMR(I).EQ.NAMA(J)) GO TO 20                              R2A00290
   10     CONTINUE                                                      R2A00300
        GO TO 40                                                        R2A00310
   20 JJ=I*(I-1)/2                                                      R2A00320
        DO 30 K=1,I                                                     R2A00330
   30     A(K,J)=R(JJ+K)                                               R2A00340
   40   CONTINUE                                                       R2A00350
C                                                                       R2A00360
      RETURN                                                            R2A00370
      END                                                               R2A00380
```

```
      SUBROUTINE R2RA (R,NR,NAM,RA,NRA,NAMA)
C                                                                       R2RA0010
C        TO COPY THE UPPER LEFT (LOWER RIGHT) PORTION OF A VECTOR       R2RA0020
C        STORED UPPER TRIANGULAR MATRIX R INTO THE LOWER RIGHT          R2RA0030
C        (UPPER LEFT) PORTION OF A VECTOR STORED TRIANGULAR             R2RA0040
C        MATRIX RA.                                                     R2RA0050
C                                                                       R2RA0060
C        R(NR*(NR+1)/2)     INPUT VECTOR STORED UPPER TRIANGULAR MATRIX  R2RA0070
C        NR                 DIMENSION OF R                              R2RA0080
C        NAM(NR)            NAMES ASSOCIATED WITH R                     R2RA0090
C        RA(NRA*(NRA+1)/2)  OUTPUT VECTOR STORED UPPER TRIANGULAR MATRIX R2RA0100
C        NRA                DIMENSION ASSOCIATED WITH RA                R2RA0110
C        NAMA(NRA)          NAMES ASSOCIATED WITH RA                    R2RA0120
C                                                                       R2RA0130
C     IF NRA=0 ON INPUT, THEN NAMA(1) SHOULD HAVE THE FIRST NAME OF THE R2RA0140
C     OUTPUT NAMELIST AND THE NUMBER OF NAMES IN NAMA IS COMPUTED.      R2RA0150
C     THE LOWER RIGHT BLOCK OF R WILL BE THE UPPER LEFT BLOCK OF RA.    R2RA0160
C                                                                       R2RA0170
C     IF NRA=LAST NAME OF THE UPPER LEFT BLOCK THAT IS TO BE MOVED,     R2RA0180
C     THEN THE UPPER BLOCK IS TO BE MOVED TO THE LOWER RIGHT POSITION.  R2RA0190
C     WHEN USED IN THIS MODE NRA=NR ON OUTPUT.                          R2RA0200
C                                                                       R2RA0210
C     THE NAMES OF THE RELOCATED BLOCK ARE ALSO MOVED.  THE RESULT      R2RA0220
C     CAN COINCIDE WITH R AND NAMA WITH NAM.                            R2RA0230
C                                                                       R2RA0240
C        COGNIZANT PERSONS:  G.J.BIERMAN/M.W.NEAD (JPL, SEPT. 1976)     R2RA0250
C                                                                       R2RA0260
      IMPLICIT DOUBLE PRECISION   (A-H,O-Z)                             R2RA0270
      DIMENSION     R(1),RA(1), NAM(1), NAMA(1)                         R2RA0280
      LOGICAL       IS                                                  R2RA0290
C                                                                       R2RA0300
      IS=.FALSE.                                                        R2RA0310
      LOCN=NAMA(1)                                                      R2RA0320
C        IS=FALSE CORRESPONDS TO MOVING UPPER LFT. CORNER OF R TO       R2RA0330
C        LOWER RT. CORNER OF RA                                         R2RA0340
      IF (NRA.EQ.0) GO TO 1                                             R2RA0350
      LOCN=NRA                                                          R2RA0360
      IS=.TRUE.                                                         R2RA0370
C        IS=TRUE CORRESPONDS TO MOVING LOWER LFT. CORNER OF R TO        R2RA0380
C        UPPER RT. CORNER OF RA                                         R2RA0390
    1 DO 3 I=1,NR                                                       R2RA0400
       IF (NAM(I).EQ.LOCN) GO TO 4                                      R2RA0410
    3 CONTINUE                                                          R2RA0420
       WRITE (6,100)                                                    R2RA0430
  100 FORMAT (1H0,20X,'NAMA(1) NOT IN NAMELIST OF R MATRIX')            R2RA0440
       RETURN                                                           R2RA0450
C                                                                       R2RA0460
    4 K=I                                                               R2RA0470
       KM1=K-1                                                          R2RA0480
      IF (IS) GO TO 15                                                  R2RA0490
C                                                                       R2RA0500
      IJS=K*(K+1)/2-1                                                   R2RA0510
       NRA=NR-K+1                                                       R2RA0520
      IJA=0                                                             R2RA0530
      KOLA=0                                                            R2RA0540
```

```
      DO 10 KOL=K,NR                                                    R2RA0550
        KOLA=KOLA+1                                                     R2RA0560
      NAMA(KOL-KM1)=NAM(KOL)                                            R2RA0570
        DO 5 IR=1,KOLA                                                  R2RA0580
          IJA=IJA+1                                                     R2RA0590
    5     RA(IJA)=R(IJS+IR)                                             R2RA0600
   10   IJS=IJS+KOL                                                     R2RA0610
      RETURN                                                            R2RA0620
C                                                                       R2RA0630
   15  IJ=K*(K+1)/2                                                     R2RA0640
       IJA=NR*(NR+1)/2                                                  R2RA0650
       L=NR-KM1                                                         R2RA0660
       KOL=K                                                            R2RA0670
          DO 25 KOLA=NR,L,-1                                            R2RA0680
          IJS=IJA                                                       R2RA0690
          NAMA(KOLA)=NAM(KOL)                                           R2RA0700
            DO 20 IR=KOLA,1,-1                                          R2RA0710
              RA(IJS)=R(IJ)                                             R2RA0720
              IJS=IJS-1                                                 R2RA0730
   20         IJ=IJ-1                                                  R2RA0740
          IJA=IJA-KOLA                                                  R2RA0750
   25     KOL=KOL-1                                                     R2RA0760
       NRA=NR                                                           R2RA0770
C                                                                       R2RA0780
      RETURN                                                            R2RA0790
      END                                                               R2RA0800
```

```
      SUBROUTINE RUDR(RIN,N,ROUT,IS)
C                                                                       RUDR0010
C     FOR N.GT.0 THIS SUBROUTINE TRANSFORMS AN UPPER TRIANGULAR VECTOR  RUDR0020
C     STORED SRIF MATRIX TO U-D FORM, AND WHEN N.LT.0 THE U-D VECTOR    RUDR0030
C     STORED ARRAY IS TRANSFORMED TO A VECTOR STORED SRIF ARRAY .       RUDR0040
C                                                                       RUDR0050
C     RIN((N+1)*(N+2)/2)   INPUT VECTOR STORED SRIF OR U-D ARRAY        RUDR0060
C     ROUT((N+1)*(N+2)/2)  OUTPUT IS THE CORRESPONDING U-C OR SRIF      RUDR0070
C                          ARRAY (RIN=ROUT IS PERMITTED)                RUDR0080
C     N                    ABS(N)= MATRIX DIMENSION                     RUDR0090
C           N.GT.0         THE (INPUT) SRIF ARRAY IS OUTPUT IN U-D FORM RUDR0100
C           N.LT.0         THE (INPUT) U-D ARRAY IS OUTPUT IN SRIF FORM RUDR0110
C     IS =  0              THERE IS NO RT. SIDE OR ESTIMATE STORED IN   RUDR0120
C                          COLUMN N+1, AND RIN NEED HAVE ONLY           RUDR0130
C                          N COLUMNS, I.E. RIN(N*(N+1)/2)               RUDR0140
C     IS =  1              THERE IS A RT. SIDE INPUT TO THE SRIF AND    RUDR0150
C                          AN ESTIMATE FOR THE U-D ARRAY. THESE RESIDE  RUDR0160
C                          IN COLUMN N+1.                               RUDR0170
C                                                                       RUDR0180
C     THIS SUBROUTINE USES SUBROUTINE UTINV                             RUDR0190
C                                                                       RUDR0200
C     COGIZANT PERSONS G.J.BIERMAN/M.W.NEAD  (JPL, FEB.1977)            RUDR0210
C                                                                       RUDR0220
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               RUDR0230
      DIMENSION RIN(1), ROUT(1)                                         RUDR0240
C                                                                       RUDR0250
      ONE= 1.0                                                          RUDR0260
      NP1= IS + ABS(N)                                                  RUDR0270
      JJ=1                              @ INITIALIZE DIAGONAL INDEX     RUDR0280
      IDIMR= NP1*(NP1 +1)/2                                             RUDR0290
      IF (IS.EQ.1) RIN(IDIMR)= - ONE                                    RUDR0300
C                                                                       RUDR0310
      IF (N.LT.0) GO TO 30                                              RUDR0320
      CALL UTINV(RIN,NP1,ROUT)                                          RUDR0330
      ROUT(1)= ROUT(1)**2                                               RUDR0340
      IF (N.EQ.1) RETURN                                                RUDR0350
      DO 20 J=2,N                                                       RUDR0360
      S=ONE/ROUT(JJ+J)                                                  RUDR0370
      ROUT(JJ+J)= ROUT(JJ+J)**2                                         RUDR0380
      JM1=J-1                                                           RUDR0390
      DO 10 I=1,JM1                                                     RUDR0400
   10 ROUT(JJ+I)= ROUT(JJ+I)*S                                          RUDR0410
   20 JJ=JJ+ J                                                          RUDR0420
      RETURN                                                            RUDR0430
C                                                                       RUDR0440
   30 N=-N                                                              RUDR0450
      ROUT(1)= SQRT(RIN(1))                                             RUDR0460
      IF(N.EQ.1) GO TO 60                                               RUDR0470
      DO 50 J=2,N                                                       RUDR0480
      ROUT(JJ+J)= SQRT(RIN(JJ+J))                                       RUDR0490
      S=ROUT(JJ+J)                                                      RUDR0500
      JM1=J-1                                                           RUDR0510
      DO 40 I=1,JM1                                                     RUDR0520
   40 ROUT(JJ+I)= RIN(JJ+I)*S                                           RUDR0530
   50 JJ=JJ+J                                                           RUDR0540
   60 CALL UTINV(ROUT,NP1,ROUT)                                         RUDR0550
C                                                                       RUDR0560
                                                                        RUDR0570
      RETURN                                                            RUDR0580
      END                                                               RUDR0590
```

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      SUBROUTINE THH(R,N,A,IA,M,SOS,NSTRT)
C                                                                       THH00010
C       THIS SUBROUTINE PERFORMS A DOUBLE PRECISION TRIANGULARIZATION   THH00020
C       OF A RECTANGULAR MATRIX INTO A SINGLY-SUBSCRIPTED ARRAY BY      THH00030
C       APPLICATION OF HOUSEHOLDER ORTHONORMAL TRANSFORMATIONS.         THH00040
C                                                                       THH00050
C       R(N*(N+3)/2) VECTOR STORED SQUARE ROOT INFORMATION MATRIX       THH00060
C                    (LAST N LOCATIONS MAY CONTAIN A RIGHT HAND SIDE)   THH00070
C       N            NUMBER OF PARAMETERS                               THH00080
C       A(IA,N+1)    MEASUREMENT MATRIX                                 THH00090
C       IA           ROW DIMENSION OF A                                 THH00100
C       M            NUMBER OF OBSERVATIONS IN THIS BATCH               THH00110
C       SOS          ACCUMULATED SUM OF SQUARES OF THE RESIDUALS        THH00120
C                    (Z-A*X(EST)**2), INCLUDES A PRIORI                 THH00130
C       NSTRT        FIRST COL OF THE INPUT A MATRIX THAT HAS A NONZERO THH00140
C                    ENTRY. IF NSTRT.LE.1, IT IS SET TO 1. THIS OPTION  THH00150
C                    IS CONVENIENT WHEN PACKING A PRIORI BY BATCHES AND THH00160
C                    THE A MATRIX HAS LEADING COLUMNS OF ZEROS.         THH00170
C                                                                       THH00180
C     ON ENTRY R CONTAINS A PRIORI SQUARE ROOT INFORMATION FILTER (SRIF)THH00190
C        ARRAY, AND ON EXIT IT CONTAINS THE A POSTERIORI (PACKED) ARRAY.THH00200
C     ON ENTRY A CONTAINS OBSERVATIONS WHICH ARE DESTROYED BY THE       THH00210
C        INTERNAL COMPUTATIONS.                                         THH00220
C     ON ENTRY IF SOS IS .LT. ZERO ,PROGRAM WILL ASSUME THERE IS NO     THH00230
C        RIGHT HAND SIDE DATA AND WILL NOT COMPUTE SOS OR USE LAST N    THH00240
C        LOCATIONS OF VECTOR R.                                         THH00250
C                                                                       THH00260
C          COGNIZANT  PERSONS G.J.BIERMAN/N.HAMATA (JPL, OCT.1975)      THH00270
C                                                                       THH00280
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               THH00290
      DIMENSION A(IA,1),R(1)                                            THH00300
      DOUBLE PRECISION SUM                                              THH00310
      DATA ZERO/0.D0/, ONE/1.D0/                                        THH00320
C                                                                       THH00330
      IF (NSTRT.LE.0) NSTRT=1                                           THH00340
      NP1=N+1                       @ NO. COLUMNS OF R                  THH00350
      IF(SOS.LT.ZERO) NP1=N         @ NO COLS. = N IF SOS.LT.0          THH00360
      KK=NSTRT*(NSTRT-1)/2                                              THH00370
      DO 100 J=NSTRT,N              @ J-TH STEP OF HOUSEHOLDER REDUCTION THH00380
      KK=KK+J                                                           THH00390
      SUM=ZERO                                                          THH00400
      DO 20 I=1,M                                                       THH00410
   20 SUM=SUM+A(I,J)**2                                                 THH00420
      IF(SUM.LE.ZERO) GO TO 100  @ IF J-TH COL. OF A.EQ.0 GO TO STEP J+1THH00430
      SUM=SUM+R(KK)**2                                                  THH00440
      SUM=DSQRT(SUM)                                                    THH00450
      IF(R(KK).GT.ZERO) SUM=-SUM                                        THH00460
      DELTA=R(KK)-SUM                                                   THH00470
      R(KK)=SUM                                                         THH00480
      BETA=ONE/(SUM*DELTA)                                              THH00490
      JJ=KK                                                             THH00500
      L=J                                                               THH00510
      J1=J+1                                                            THH00520
C        ** READY TO APPLY J-TH HOUSEHOLDER TRANS.                      THH00530
      DO 40 K=J1,NP1                                                    THH00540
```

```
      JJ=JJ+L                                                          THH00550
      L=L+1                                                            THH00560
      SUM=DELTA*R(JJ)                                                  THH00570
      DO 30 I=1,M                                                      THH00580
   30 SUM=SUM+A(I,J)*A(I,K)                                            THH00590
      IF(SUM.EQ.ZERO) GO TO 40                                         THH00600
      SUM=SUM*BETA                                                     THH00610
      R(JJ)=R(JJ)+SUM*DELTA                                            THH00620
      DO 35 I=1,M                                                      THH00630
   35 A(I,K)=A(I,K)+SUM*A(I,J)                                         THH00640
   40 CONTINUE                                                         THH00650
  100 CONTINUE                                                         THH00660
      IF(SOS.LT.ZERO) RETURN                                           THH00670
C                                                                      THH00680
C     CALCULATE SOS                                                    THH00690
C                                                                      THH00700
      SUM=ZERO                                                         THH00710
      DO 110 I=1,M                                                     THH00720
  110 SUM=SUM+A(I,NP1)**2                                              THH00730
      SOS=DSQRT(SOS**2+SUM)                                            THH00740
C                                                                      THH00750
      RETURN                                                           THH00760
      END                                                              THH00770
```

```
      SUBROUTINE TRIMAT (A,N,CAR,TEXT,NCHAR,NAMES)
C                                                                       TRIM0010
C        TO DISPLAY A VECTOR STORED UPPER TRIANGULAR MATRIX IN A        TRIM0020
C        TWO-DIMENSIONAL TRIANGULAR FORMAT                              TRIM0030
C                                                                       TRIM0040
C        A(N*(N+1)/2) VECTOR CONTAINING UPPER TRIANGULAR MATRIX   (DP)  TRIM0050
C        N            DIMENSION OF MATRIX                          (I)  TRIM0060
C        CAR(N)       PARAMETER NAMES                              (I)  TRIM0070
C        TEXT( )      AN ARRAY OF FIELDATA CHARACTERS TO BE PRINTED AS  TRIM0080
C                     A TITLE PRECEDING THE MATRIX                      TRIM0090
C        NCHAR        NUMBER OF CHARACTERS, INCLUDING SPACES, THAT      TRIM0100
C                     ARE TO BE PRINTED IN TEXT( )                      TRIM0110
C                     ABS(NCHAR).LE.126. NCHAR NEGATIVE IS USED         TRIM0120
C                     TO AVOID SKIPPING TO A NEW PAGE TO START          TRIM0130
C                     PRINTING                                          TRIM0140
C        NAMES        TRUE TO PRINT PARAMETER NAMES                     TRIM0150
C                                                                       TRIM0160
C         COGNIZANT PERSONS:  G.J.BIERMAN/M.W.NEAD (JPL, OCT.1975)      TRIM0170
C                                                                       TRIM0180
      DOUBLE PRECISION  A(N)                                            TRIM0190
      INTEGER CAR(N), TEXT(1), L(7), LIST(7)                            TRIM0200
      LOGICAL NAMES                                                     TRIM0210
      INTEGER  V(4),VFMT(7)                                             TRIM0220
      DATA  V/'(2X,','A6,1X,',',  ','D17.8)'/,                         TRIM0230
     1 VFMT/'7','017X,6','034X,5','051X,4','068X,3','085X,2','102X,1'/ TRIM0240
C                                                                       TRIM0250
C        M1,M2     ROW LIMITS FOR EACH PRINT SEQUENCE                   TRIM0260
C        N1,M2     COL LIMITS FOR EACH LINE OF PRINT                    TRIM0270
C        L(I)      LOC OF EACH COLUMN IN A ROW                          TRIM0280
C        KT        ROW COUNTER                                          TRIM0290
C        KP        PRINT COUNTER                                        TRIM0300
C                                                                       TRIM0310
C  * * * * *   INITIALIZE COUNTERS                                     TRIM0320
C                                                                       TRIM0330
      M1=1                                                              TRIM0340
      M2=7                                                              TRIM0350
      N1=1                                                              TRIM0360
      KT=0                                                              TRIM0370
      KP=0                                                              TRIM0380
      IF (.NOT.NAMES) V(2)='I5,2X'                                      TRIM0390
C                                                                       TRIM0400
      NC=IABS(NCHAR)/6                                                  TRIM0410
      IF (MOD(NCHAR,6).NE.0) NC=NC+1                                    TRIM0420
      IF (NCHAR.GE.0) WRITE (6,200) (TEXT(I),I=1,NC)                    TRIM0430
      IF (NCHAR.LT.0) WRITE (6,205) (TEXT(I),I=1,NC)                    TRIM0440
   10 IF (M2.GT.N) M2=N                                                 TRIM0450
      IF (.NOT.NAMES) GO TO 20                                          TRIM0460
      WRITE (6,210) (CAR(I),I=N1,M2)                                    TRIM0470
      GO TO 40                                                          TRIM0480
   20 M=N1                                                              TRIM0490
      L2=M2-N1+1                                                        TRIM0500
      DO 30 I=1,L2                                                      TRIM0510
        LIST(I)=M                                                       TRIM0520
   30   M=M+1                                                          TRIM0530
      WRITE (6,220) (LIST(I),I=1,L2)                                    TRIM0540
```

```
   40 CONTINUE                                                          TRIM0550
C    * * * * * *                                                       TRIM0560
       DO 190 IC=M1,M2                                                 TRIM0570
         K=1                                                           TRIM0580
         IF (IC.LE.(KT*7)) GO TO 60                                    TRIM0590
         JJ=0                                                          TRIM0600
         DO 50 J=1,IC                                                  TRIM0610
   50      JJ=JJ+J                                                     TRIM0620
         L(K)=JJ                                                       TRIM0630
         I1=IC-KT*7                                                    TRIM0640
         IF (I1.EQ.7) GO TO 90                                         TRIM0650
         GO TO 70                                                      TRIM0660
   60    CONTINUE                                                      TRIM0670
C                                                                      TRIM0680
         I1=1                                                          TRIM0690
         L(K)=L(K)+1                                                   TRIM0700
   70    CONTINUE                                                      TRIM0710
         DO 80 I=I1,6                                                  TRIM0720
           K=K+1                                                       TRIM0730
           II=I+KT*7                                                   TRIM0740
   80      L(K)=L(K-1)+II                  @ OBTAIN COL INDEX FOR ROW  TRIM0750
   90    CONTINUE                                                      TRIM0760
C                                                                      TRIM0770
         I2=MIN0(8,(M2+1-KT*7))-I1                                     TRIM0780
         V(3)=VFMT(I1)                                                 TRIM0790
         IF (.NOT.NAMES) GO TO 180                                     TRIM0800
         WRITE (6,V) CAR(IC),(A(L(I)),I=1,I2)                          TRIM0810
         GO TO 190                                                     TRIM0820
  180    WRITE (6,V) IC,(A(L(I)),I=1,I2)                               TRIM0830
  190    CONTINUE                                                      TRIM0840
       IF (M2.EQ.N) RETURN                                             TRIM0850
       N1=M2+1                                                         TRIM0860
       M2=M2+7                                                         TRIM0870
       KT=KT+1                                                         TRIM0880
       KP=KP+1                                                         TRIM0890
       IF (KP.LT.3) GO TO 10                                           TRIM0900
       WRITE (6,200) (TEXT(I),I=1,NC)                                  TRIM0910
       GO TO 10                                                        TRIM0920
C                                                                      TRIM0930
  200 FORMAT (1H1,2X,21A6)             @ TITLE                         TRIM0940
  205 FORMAT (1H0,2X,21A6)             @ TITLE                         TRIM0950
  210 FORMAT (1H0,5X,7(11X,A6))         @ HORIZONTAL NAMES             TRIM0960
  220 FORMAT (1H0,3X,7(11X,I6))                                        TRIM0970
C                                                                      TRIM0980
       END                                                             TRIM0990
```

```
      SUBROUTINE TTHH(R,RA,N)
C                                                                       TTHH0010
C     THIS SUBROUTINE COMBINES TWO SINGLE SUBSCRIPTED SRIF ARRAYS       TTHH0020
C     USING HOUSEHOLDER ORTHOGONAL TRANSFORMATIONS                      TTHH0030
C                                                                       TTHH0040
C       R(N*(N+1)/2)   VECTOR STORED SRIF ARRAY.                        TTHH0050
C       RA(N*(N+1)/2)  THE SECOND VECTOR STORED SRIF ARRAY              TTHH0060
C       N              DIMENSION OF THE ESTIMATED PARAMETER VECTOR.     TTHH0070
C                      A NEGATIVE VALUE FOR N IS USED TO NOTE THAT      TTHH0080
C                      R AND RA HAVE RT. HAND SIDES INCLUDED AND        TTHH0090
C                      HAVE DIM=ABSN*(ABSN+3)/2.                        TTHH0100
C                                                                       TTHH0110
C       ON EXIT RA IS CHANGED AND R CONTAINS THE RESULTING SRIF ARRAY   TTHH0120
C                                                                       TTHH0130
C         COGNIZANT PERSONS G.J.BIERMAN/M.W.NEAD (JPL, JAN.1976)        TTHH0140
C                                                                       TTHH0150
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)                                TTHH0160
      DIMENSION RA(1), R(1)                                             TTHH0170
C     DOUBLE PRECISION SUM  @ FOR USE IN SINGLE PRECISION VERSION       TTHH0180
C                                                                       TTHH0190
      ZERO=0.                                                           TTHH0200
      ONE=1.                                                            TTHH0210
      NP1=N                                                             TTHH0220
      IF (N.GT.0) GO TO 10                                              TTHH0230
      N=-N                                                              TTHH0240
      NP1=N+1                                                           TTHH0250
   10   IJS=1                  @ IJ(START)                               TTHH0260
        KK=0                                                            TTHH0270
        DO 100 J=1,N           @ J-TH STEP OF HOUSEHOLDER REDUCTION      TTHH0280
          KK=KK+J                                                       TTHH0290
        SUM=R(KK)**2                                                    TTHH0300
        DO 20 I=IJS,KK                                                  TTHH0310
   20   SUM=SUM+RA(I)**2                                                TTHH0320
      IF (SUM.LE.ZERO) GO TO 100                                        TTHH0330
        SUM=SQRT(SUM)                                                   TTHH0340
        IF (R(KK).GT.ZERO) SUM=-SUM                                     TTHH0350
        DELTA=R(KK)-SUM                                                 TTHH0360
        R(KK)=SUM                                                       TTHH0370
        BETA=ONE/(SUM*DELTA)                                            TTHH0380
        JJ=KK                                                           TTHH0390
        L=J                                                             TTHH0400
        JP1=J+1                                                         TTHH0410
        IKS=KK+1                                                        TTHH0420
C           * * * J-TH HOUSEHOLDER TRANS. DEFINED                       TTHH0430
C                 40 LOOP APPLIES TRANSFORM. TO COLS. J+1 TO NP1        TTHH0440
        DO 40 K=JP1,NP1                                                 TTHH0450
        JJ=JJ+L                                                         TTHH0460
        L=L+1                                                           TTHH0470
        IK=IKS                                                          TTHH0480
        SUM=DELTA*R(JJ)                                                 TTHH0490
        DO 30 I=IJS,KK                                                  TTHH0500
        SUM=SUM+RA(IK)*RA(I)                                            TTHH0510
   30   IK=IK+1                                                         TTHH0520
        IF (SUM.EQ.ZERO) GO TO 40                                       TTHH0530
        SUM=SUM*BETA                                                    TTHH0540
```

```
      R(JJ)=R(JJ)+SUM*DELTA                                          TTHH0550
      IK=IKS                                                         TTHH0560
      DO 35 I=IJS,KK                                                 TTHH0570
      RA(IK)=RA(IK)+SUM*RA(I)                                        TTHH0580
   35 IK=IK+1                                                        TTHH0590
   40  IKS=IKS+K                                                     TTHH0600
  100 IJS=KK+1                                                       TTHH0610
C                                                                    TTHH0620
      RETURN                                                         TTHH0630
      END                                                            TTHH0640
```

```
      SUBROUTINE TZERO (R,N,IS,IF)
C                                                                       TZER0010
C        TO ZERO OUT ROWS IS (ISTART) TO IF (IFINAL) OF A VECTOR        TZER0020
C        STORED UPPER TRIANGULAR MATRIX                                 TZER0030
C                                                                       TZER0040
C        R(N*(N+1)/2)  INPUT VECTOR STORED UPPER TRIANGULAR MATRIX      TZER0050
C        N             DIMENSION OF R                                   TZER0060
C        IS            FIRST ROW OF R THAT IS TO BE SET TO ZERO         TZER0070
C        IR            LAST ROW OF R THAT IS TO BF SET TO ZERO          TZER0080
C                                                                       TZER0090
C        COGNIZANT PERSONS:  G.J.BIERMAN/C.F.PETERS (JPL, NOV. 1975)    TZER0100
C                                                                       TZER0110
      IMPLICIT DOURLE PRECISION (A-H,O-Z)                               TZER0120
      DIMENSION R(1)                                                    TZER0130
C                                                                       TZER0140
      ZERO=0.0                                                          TZER0150
        IJS=IS*(IS-1)/2                                                 TZER0160
        DO 10 I=IS,IF                                                   TZER0170
        IJS=IJS+I                                                       TZER0180
        IJ=IJS                                                          TZER0190
        DO 10 J=I,N                                                     TZER0200
        R(IJ)=ZERO                                                      TZER0210
        IJ=IJ+J                                                         TZER0220
   10 CONTINUE                                                          TZER0230
C                                                                       TZER0240
        RETURN                                                          TZER0250
        END                                                             TZER0260
```

```
      SUBROUTINE UDMES (U,N,R,A,G,ALPHA)                                UDMES010
C                                                                       UDMES020
C       COMPUTES ESTIMATE AND U-D MEASUREMENT UPDATED                   UDMES030
C       COVARIANCE, P=UDU**T                                            UDMES040
C                                                                       UDMES050
C       *** INPUTS ***                                                  UDMES060
C                                                                       UDMES070
C       U       UPPER TRIANGULAR MATRIX, WITH D ELEMENTS STORED AS THE  UDMES080
C               DIAGONAL. U IS VECTOR STORED AND CORRESPONDS TO THE     UDMES090
C               A PRIORI COVARIANCE. IF STATE ESTIMATES ARE COMPUTED,   UDMES100
C               THE LAST COLUMN OF U CONTAINS X.                        UDMES110
C       N       DIMENSION OF THE STATE ESTIMATE.                        UDMES120
C       R       MEASUREMENT VARIANCE                                    UDMES130
C       A       VECTOR OF MEASUREMENT COEFFICIENTS, IF DATA THEN A(N+1)=ZUDMES140
C       ALPHA   IF ALPHA LESS THAN ZERO NO ESTIMATES ARE COMPUTED       UDMES150
C               (AND X AND Z NEED NOT BE INCLUDED)                      UDMES160
C                                                                       UDMES170
C       *** OUTPUTS ***                                                 UDMES180
C                                                                       UDMES190
C       U        UPDATED, VECTOR STORED FACTORS AND ESTIMATE AND        UDMES200
C                U((N+1)(N+2)/2) CONTAINS (Z-A**T*X)                    UDMES210
C                                                                       UDMES220
C       ALPHA    INNOVATIONS VARIANCE OF THE MEASUREMENT RESIDUAL       UDMES230
C       G        VECTOR OF UNWEIGHTED KALMAN GAINS, K=G/ALPHA           UDMES240
C       A        CONTAINS U**TA AND (Z-A**T*X)/ALPHA                    UDMES250
C                                                                       UDMES260
C          COGNIZANT PERSONS: G.J. BIERMAN/M.W. NEAD (JPL, SEPT.1976)   UDMES270
C                                                                       UDMES280
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                               UDMES290
      DIMENSION U(1), A(1), G(1)                                        UDMES300
      DOUBLE PRECISION SUM                                              UDMES310
      LOGICAL IEST                                                      UDMES320
C                                                                       UDMES330
      ZERO=0.0                                                          UDMES340
      IEST=.FALSE.                                                      UDMES350
      ONE=1.                                                            UDMES360
      NP1=N+1                                                           UDMES370
      NTOT=N*NP1/2                                                      UDMES380
      IF (ALPHA.LT.ZERO) GO TO 3                                        UDMES390
      SUM=A(NP1)                                                        UDMES400
      DO 1 J=1,N                                                        UDMES410
    1     SUM=SUM-A(J)*U(NTOT+J)                                        UDMES420
      U(NTOT+NP1)=SUM                        @ Z=Z-A**T*X               UDMES430
      IEST=.TRUE.                                                       UDMES440
C                                                                       UDMES450
    3 KJ=NTOT                                                           UDMES460
      DO 10 J=N,2,-1                                                    UDMES470
          SUM=A(J)                                                      UDMES480
          JM1=J-1                                                       UDMES490
          DO 5 K=JM1,1,-1                                               UDMES500
            KJ=KJ-1                                                     UDMES510
    5       SUM=SUM+U(KJ)*A(K)                                          UDMES520
          A(J)=SUM                                                      UDMES530
          KJ=KJ-1                                                       UDMES540
   10   G(J)=SUM*U(KJ+J)                                                UDMES550
```

```
      G(1)=U(1)*A(1)                                                    UDMES560
C          A=U**T*A AND G=D*(U**T*A)                                    UDMES570
C                                                                       UDMES580
      SUM=R+G(1)*A(1)                      @ SUM(1)                     UDMES590
C     GAMMA=0                                @ FOR R=0 CASE             UDMES600
C     IF (G(1).EQ.ZERO) GO TO 11             @ FOR R=0 CASE             UDMES610
      GAMMA=ONE/SUM                                                     UDMES620
      U(1)=U(1)*R*GAMMA                    @ D(1)                       UDMES630
C                                                                       UDMES640
   11 KJ=2                                                              UDMES650
      DO 20 J=2,N                                                       UDMES660
          BETA=SUM                         @ BETA=SUM(J-1)              UDMES670
          SUM=SUM+G(J)*A(J)                @ SUM(J)                     UDMES680
          P=-A(J)*GAMMA                    @ P=-F(J)*(1/SUM(J-1)) EQN(21) UDMES690
          JM1=J-1                                                       UDMES700
          DO 15 K=1,JM1                                                 UDMES710
            S=U(KJ)                                                     UDMES720
            U(KJ)=S+P*G(K)                 @ EQN(22)                    UDMES730
            G(K)=G(K)+G(J)*S               @ EQN(23)                    UDMES740
   15             KJ=KJ+1                                               UDMES750
C     IF (G(J).EQ.ZERO) GO TO 20             @ FOR R=0 CASE             UDMES760
          GAMMA=ONE/SUM                    @ GAMMA=1/SUM(J)             UDMES770
          U(KJ)=U(KJ)*BETA*GAMMA           @ D(J)      EQN(19)          UDMES780
   20 KJ=KJ+1                                                           UDMES790
      ALPHA=SUM                                                         UDMES800
C                                                                       UDMES810
C          EQN. NOS. REFER TO BIERMAN'S 1975 CDC PAPER, PP. 337-346.    UDMES820
C                                                                       UDMES830
      IF (.NOT.IEST) RETURN                                             UDMES840
      A(NP1)=U(NTOT+NP1)*GAMMA                                          UDMES850
      DO 30 J=1,N                                                       UDMES860
   30   U(NTOT+J)=U(NTOT+J)+G(J)*A(NP1)                                  UDMES870
C                                                                       UDMES880
      RETURN                                                            UDMES890
      END                                                               UDMES900
```

```
C
      SUBROUTINE UD2COV (UIN,POUT,N)                                        UD2C0010
C                                                                           UD2C0020
C     TO OBTAIN A COVARIANCE FROM ITS U-D FACTORIZATION. BOTH MATRICES      UD2C0030
C     ARE VECTOR STORED AND THE OUTPUT COVARIANCE CAN OVERWRITE THE         UD2C0040
C     INPUT U-D ARRAY. UIN=U-D IS RELATED TO POUT VIA POUT=UDU(**T)         UD2C0050
C                                                                           UD2C0060
C     UIN(N*(N+1)/2)  INPUT U-D FACTORS, VECTOR STORED WITH THE D           UD2C0070
C                     ENTRIES STORED ON THE DIAGONAL OF UIN                 UD2C0080
C     POUT(N*(N+1)/2) OUTPUT COVARIANCE, VECTOR STORED.                     UD2C0090
C                     (POUT=UIN IS PERMITTED)                               UD2C0100
C     N               DIMENSION OF THE MATRICES INVOLVED                    UD2C0110
C                                                                           UD2C0120
C     COGNIZANT PERSONS: G.J.BIERMAN/M.W.NEAD  (JPL, FEB. 1977)             UD2C0130
C                                                                           UD2C0140
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                                   UD2C0150
C                                                                           UD2C0160
      DIMENSION   UIN(1),   POUT(1)                                         UD2C0170
C                                                                           UD2C0180
      POUT(1)=UIN(1)                                                        UD2C0190
      JJ=1                                                                  UD2C0200
      DO 20 J=2,N                                                           UD2C0210
        JJL=JJ                          @ (J-1,J-1)                         UD2C0220
        JJ=JJ+J                                                             UD2C0230
        POUT(JJ)=UIN(JJ)                                                    UD2C0240
        S=POUT(JJ)                                                          UD2C0250
        II=0                                                                UD2C0260
        JM1=J-1                                                             UD2C0270
        DO 20 I=1,JM1                                                       UD2C0280
          II=II+I                                                           UD2C0290
          ALPHA=S*UIN(JJL+I)            @ JJL+I=(I,J)                       UD2C0300
          IK=II                                                             UD2C0310
          DO 10 K=I,JM1                                                     UD2C0320
            POUT(IK)=POUT(IK)+ALPHA*UIN(JJL+K)     @ JJL+K=(K,J)            UD2C0330
   10       IK=IK+K                                                         UD2C0340
   20   POUT(JJL+I)=ALPHA                                                  UD2C0350
C                                                                           UD2C0360
      RETURN                                                                UD2C0370
      END                                                                   UD2C0380
```

```
C
      SUBROUTINE UD2SIG(U,N,SIG,TEXT,NCT)                                UD2SI010
C                                                                        UD2SI020
C     COMPUTE STANDARD DEVIATIONS (SIGMAS) FROM U-D COVARIANCE FACTORS   UD2SI030
C                                                                        UD2SI040
C     U(N*(N+1)/2)  INPUT VECTOR STORED ARRAY CONTAINING THE U-D         UD2SI050
C                   FACTORS. THE D (DIAGONAL) ELEMENTS ARE STORED        UD2SI060
C                   ON THE DIAGONAL                                      UD2SI070
C     SIG(N)        VECTOR OF OUTPUT STANDARD DEVIATIONS                 UD2SI080
C     TEXT( )       ARRAY OF FIELDATA CHARACTERS TO BE PRINTED           UD2SI090
C                   PRECEDING THE VECTOR OF SIGMAS                       UD2SI100
C     NCT           NUMBER OF CHARACTERS IN TEXT, 0.LE.NCT.LE.126        UD2SI110
C                   IF NCT=0, NO SIGMAS ARE PRINTED                      UD2SI120
C                                                                        UD2SI130
C     COGNIZANT PERSONS: G.J.BIERMAN/M.W.NEAD   (JPL, FEB. 1977)         UD2SI140
C                                                                        UD2SI150
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                                UD2SI160
      INTEGER TEXT(1)                                                    UD2SI170
      DIMENSION U(1), SIG(1)                                             UD2SI180
C                                                                        UD2SI190
      JJ=1                                                               UD2SI200
      SIG(1)=U(1)                                                        UD2SI210
      DO 10 J=2,N                                                        UD2SI220
        JJL=JJ                          @ (J-1,J-1)                      UD2SI230
        JJ=JJ+J                                                          UD2SI240
        S=U(JJ)                                                          UD2SI250
        SIG(J)=S                                                         UD2SI260
        JM1=J-1                                                          UD2SI270
        DO 10 I=1,JM1                                                    UD2SI280
   10     SIG(I)=SIG(I)+S*U(JJL+I)**2                                     UD2SI290
C                                                                        UD2SI300
C           WE NOW HAVE VARIANCES                                        UD2SI310
C                                                                        UD2SI320
      DO 20 J=1,N                                                        UD2SI330
   20   SIG(J)=SQRT(SIG(J))                                             UD2SI340
      IF (NCT.EQ.0) GO TO 30                                             UD2SI350
      NC=NCT/6                                                           UD2SI360
      IF (MOD(NC,6).NE.0) NC=NC+1                                        UD2SI370
      WRITE (6,40) (TEXT(I),I=1,NC)                                      UD2SI380
      WRITE (6,50) (SIG(I),I=1,N)                                        UD2SI390
   30 RETURN                                                             UD2SI400
C                                                                        UD2SI410
   40 FORMAT (1H0,2X,21A6)                                               UD2SI420
   50 FORMAT (1H0,(6D18.10))                                             UD2SI430
      END                                                                UD2SI440
```

```
      SUBROUTINE UTINV(RIN,N,ROUT)
C                                                                       UTINV010
C        TO INVERT AN UPPER TRIANGULAR VECTOR STORED MATRIX AND STORE   UTINV020
C        THE RESULT IN VECTOR FORM. THE ALGORITHM IS SO ARRANGED THAT   UTINV030
C        THE RESULT CAN OVERWRITE THE INPUT.                            UTINV040
C        IN ADDITION TO  SOLVE RX=Z, SET RIN(N*(N+1)/2+1)=Z(1), ETC.,   UTINV050
C        AND SET RIN((N+1)*(N+2)/2)=-1. CALL THE SUBROUTINE USING N+1   UTINV060
C        INSTEAD OF N. ON RETURN THE FIRST N ENTRIES OF COLUMN N+1      UTINV070
C        WILL CONTAIN X.                                                UTINV080
C                                                                       UTINV090
C        RIN(N*(N+1)/2)  INPUT VECTOR STORED UPPER TRIANGULAR MATRIX    UTINV100
C        N               MATRIX DIMENSION                               UTINV110
C        ROUT(N*(N+1)/2) OUTPUT VECTOR STORED UPPER TRIANGULAR MATRIX   UTINV120
C                        INVERSE                                        UTINV130
C                                                                       UTINV140
C        COGNIZANT PERSONS:  G.J.BIERMAN/J.ELLIS (JPL, SEPT. 1976)      UTINV150
C                                                                       UTINV160
      DOUBLE PRECISION  RIN(1), ROUT(1), WORK, ONE, ZERO ,DIN           UTINV170
      DATA  ONE/1.0D0/,ZERO/ 0.0D0/                                     UTINV180
      IPV =   N*(N+1)/2                                                 UTINV190
      IN  = IPV                                                         UTINV200
      DO 6    I=1,N                                                     UTINV210
      IF (RIN(IPV).NE.ZERO) GO TO 1                                     UTINV220
      WRITE (6,10) I                                                    UTINV230
      RETURN                                                            UTINV240
    1 DIN   =         ONE/ RIN(IPV)                                     UTINV250
      ROUT( IPV )  = DIN                                                UTINV260
      MIN =N                                                            UTINV270
      KEND = I-1                                                        UTINV280
      LANF  =   N - KEND                                                UTINV290
      IF (I.EQ.1) GO TO 5                                               UTINV300
    2 J= IN                                                             UTINV310
C                                                                       UTINV320
C             INITIALIZE  ROW LOOP                                      UTINV330
C                                                                       UTINV340
      DO 4   K=1,KEND                                                   UTINV350
      WORK =ZERO                                                        UTINV360
      MIN= MIN -  1                                                     UTINV370
      LIN= IPV                                                          UTINV380
      LOT= J                                                            UTINV390
C                                                                       UTINV400
C            START  INNER LOOP                                          UTINV410
C                                                                       UTINV420
      DO 3   L=LANF, MIN                                                UTINV430
      LIN= LIN+L                                                        UTINV440
      LOT= LOT+1                                                        UTINV450
    3 WORK = WORK  + RIN(LIN)* ROUT(LOT)                                UTINV460
      ROUT(J)  =   -WORK* DIN                                           UTINV470
    4 J=  J- MIN                                                        UTINV480
    5 IPV =  IPV -MIN                                                   UTINV490
    6 IN= IN -1                                                         UTINV500
      RETURN                                                            UTINV510
   10 FORMAT (1H0,10X,'UTINV DIAGONAL',I4,'IS ZERO')                    UTINV520
      END                                                               UTINV530
```

```
C
      SUBROUTINE  UTIROW (RIN,N,ROUT,NRY)                                   UTIR0010
C                                                                           UTIR0020
C     TO COMPUTE THE INVERSE OF AN UPPER TRIANGULAR (VECTOR STORED)         UTIR0030
C     MATRIX WHEN THE LOWER PORTION OF THE INVERSE IS GIVEN                 UTIR0040
C                                                                           UTIR0050
C         ON INPUT:                                                         UTIR0060
C                                                                           UTIR0070
C           RX   RXY              *     *                    RX   RXY       UTIR0080
C     RIN=                  ROUT=           WHERE     R=                    UTIR0090
C           *    *                0   RY**-1                 0    RY       UTIR0100
C                                                                           UTIR0110
C         ON OUTPUT: RIN IS UNCHANGED AND ROUT=R**-1                        UTIR0120
C        * THE RESULT CAN OVER-WRITE THE INPUT (I.E. RIN=ROUT)              UTIR0130
C                                                                           UTIR0140
C     RIN(N*(N+1)/2)     INPUT VECTOR STORED TRIANGULAR MATRIX              UTIR0150
C                        THE BOTTOM NRY ROWS ARE IGNORED                    UTIR0160
C     N                  MATRIX DIMENSION                                   UTIR0170
C     ROUT(N*(N+1)/2)    OUTPUT VECTOR STORED MATRIX. ON INPUT THE          UTIR0180
C                        BOTTOM NRY ROWS CONTAIN THE LOWER PORTION          UTIR0190
C                        OF R**-1. ON OUTPUT ROUT=R**-1                     UTIR0200
C     NRY                DIMENSION OF LOWER (ALREADY INVERTED)              UTIR0210
C                        TRIANGULAR R. IF NRY=0, ORDINARY MATRIX            UTIR0220
C                        INVERSION RESULTS.                                 UTIR0230
C                                                                           UTIR0240
C     COGNIZANT PERSONS:   G.J.BIERMAN/M.W.NEAD   (JPL MARCH 1977)          UTIR0250
C                                                                           UTIR0260
      DOUBLE PRECISION   RIN(1), ROUT(1), SUM, ZERO, ONE, DINV              UTIR0270
      DATA   ONE/1.D0/, ZERO/0.D0/                                          UTIR0280
C                                                                           UTIR0290
C         INITIALIZATION                                                    UTIR0300
C                                                                           UTIR0310
      NR=N*(N+1)/2                  @ NO. ELEMENTS IN R                   UTIR0320
      ISTRT=N-NRY                   @ FIRST ROW TO BE INVERTED            UTIR0330
      IRLST=ISTRT+1                 @ IRLST=PREVIOUS IROW INDEX           UTIR0340
      II=ISTRT*IRLST/2              @ II=DIAGONAL                         UTIR0350
      DO 40 IROW=ISTRT,1,-1                                                 UTIR0360
        IF (RIN(II).NE.ZERO) GO TO 10                                       UTIR0370
        WRITE (6,50) IROW                                                   UTIR0380
        RETURN                                                              UTIR0390
   10   DINV=ONE/RIN(II)                                                    UTIR0400
        ROUT(II)=DINV                                                       UTIR0410
        KJS=NR+IROW                 @ KJ(START)                           UTIR0420
        IKS=II+IROW                 @ IK(START)                           UTIR0430
C                                                                           UTIR0440
        IF (IRLST.GT.N) GO TO 35                                            UTIR0450
        DO 30 J=N,IRLST,-1                                                  UTIR0460
          KJS=KJS-J                                                         UTIR0470
          SUM=ZERO                                                          UTIR0480
          IK=IKS                                                            UTIR0490
          KJ=KJS                                                            UTIR0500
C                                                                           UTIR0510
          DO 20 K=IRLST,J                                                   UTIR0520
            KJ=KJ+1                                                         UTIR0530
            SUM=SUM+RIN(IK)*ROUT(KJ)                                        UTIR0540
```

```
   20       IK=IK+K                                                    UTIR0550
C                                                                     UTIR0560
   30     ROUT(KJS)=-SUM*DINV                                         UTIR0570
   35   IRLST=IROW                                                    UTIR0580
   40   II=II-IROW                                                    UTIR0590
      RETURN                                                          UTIR0600
   50 FORMAT (1H0,10X,'RIN DIAGONAL',I4,'IS ZERO')                    UTIR0610
      END                                                             UTIR0620
```

```
C
      SUBROUTINE WGS (W,IMAXW,IW,JW,D,U,V)                                  WGS00010
C     MODIFIED GRAMM-SCHMIDT ALGORITHM FOR REDUCING WDW(**T) TO UnU(**T)WGS00020
C     FORM WHERE U IS A VECTOR STORED TRIANGULAR MATRIX WITH THE          WGS00030
C     RESULTING D ELEMENTS STORE ON THE DIAGONAL                          WGS00040
C                                                                         WGS00050
C     W(IW,JW)        INPUT MATRIX TO BE REDUCED TO TRIANGULAR FORM.      WGS00060
C                     THIS MATRIX IS DESTROYED BY THE CALCULATION         WGS00070
C                     IW.LE.IMAXW.                                        WGS00080
C     D(IW)           VECTOR OF NON-NEGATIVE WEIGHTS FOR THE              WGS00090
C                     ORTHOGONALIZATION PROCESS. THE D'S ARE UNCHANGED    WGS00100
C                     BY THE CALCULATION.                                 WGS00110
C     U(IW*(IW+1)/2) OUTPUT UPPER TRIANGULAR VECTOR STORED OUTPUT         WGS00120
C     V(JW)           WORK VECTOR                                         WGS00130
C                                                                         WGS00140
C                     (SEE BOOK:                                          WGS00150
C     ' FACTORIZATION METHODS FOR DISCRETE SEQUENTIAL ESTIMATION ',       WGS00160
C                      BY G.J.BIERMAN)                                    WGS00170
C      ESTIMATION                                                         WGS00180
C                                                                         WGS00190
C     COGNIZANT PERSONS: G.J.BIERMAN/M.W.NEAD   (JPL, MARCH 1977)        WGS00200
C                                                                         WGS00210
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                                 WGS00220
      DIMENSION W(IMAXW,1), D(1), U(1), V(1)                              WGS00230
C                                                                         WGS00240
      Z=0.0                                                               WGS00250
      ONE=1.0                                                             WGS00260
      DO 100 J=IW,1,-1                                                    WGS00270
        SUM=Z                                                             WGS00280
        DO 40 K=1,JW                                                      WGS00290
          V(K)=W(J,K)                                                     WGS00300
          U(K)=D(K)*V(K)              @U HERE IS USED AS A WORK VECTOR    WGS00310
   40     SUM=V(K)*U(K)+SUM                                              WGS00320
        W(J,J)=SUM                     @ EQ.(4.9) OF BOOK, NEW D(J)       WGS00330
        IF (J.EQ.1) GO TO 100                                             WGS00340
        DINV=Z                                                            WGS00350
        IF (SUM.GT.Z) DINV=ONE/SUM                                        WGS00360
        JM1=J-1                                                           WGS00370
        DO 70 K=1,JM1                                                     WGS00380
          SUM=Z                                                           WGS00390
          DO 50 I=1,JW                                                    WGS00400
   50       SUM=W(K,I)*U(I)+SUM                                          WGS00410
          SUM=SUM*DINV                                                    WGS00420
C                                                                         WGS00430
          DO 60 I=1,JW                                                    WGS00440
   60       W(K,I)=W(K,I)-SUM*V(I)                                       WGS00450
   70     W(J,K)=SUM                  @ EQ.(4.10) OF BOOK                WGS00460
  100   CONTINUE                     @ U(K,J) STORED IN W(J,K)           WGS00470
C                                                                         WGS00480
C         THE LOWER PART OF W IS U TRANSPOSE                              WGS00490
C                                                                         WGS00500
      IJ=0                                                                WGS00510
      DO 110 J=1,IW                                                       WGS00520
        DO 110 I=1,J                                                      WGS00530
          IJ=IJ+1                                                         WGS00540
  110     U(IJ)=W(J,I)                                                    WGS00550
C                                                                         WGS00560
      RETURN                                                              WGS00570
      END                                                                 WGS00580
```

References

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[2] JPL FORTRAN V Subprogram Directory, JPL Internal Document 1845-23, Rev. A., Feb. 1, 1975.

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